MIDTERM (MATH 180, SPRING 2014)

Your Name: ____

Date: ____

The rules:
You are allowed to use only this paper and pen/pencil. No calculators.
No books, no notebooks, no scratch paper, no web access. You MUST write your name.
You MUST simplify completely and BOX all answers. Except for the last problem,
you MUST write out your logical reasoning and/or proof in full.
You have exactly 50 minutes.

Points:

1	
2	
3	
4	
5	

____

Total: **(out of 100)**

Problem 1. (20 points, 5 points each part)

In each case, compute:

a) the number of permutations of $\{1, 2, 3, 4, 5\}$ with exactly one fixed point.

b) the number of 5-subsets of $\{1, 2, \ldots, 10\}$ which contain $2, 3$ and do not contain 5.

c) the number of (shortest) grid walks from $(1, 2)$ to $(5, 4)$.

d) the number of subgraphs of $K_{6,6}$ isomorphic to $K_{2,3}$.

Problem 2. (20 points, 10 points each part)

a) Suppose 200 hunters are shooting independently at random at 100 rabbits. Compute the expected number of surviving rabbits. Is it more than 10?

b) Consider the set S of all subgraphs $K_{10,10}$ which have all 20 vertices. What is the total number of edges graphs in S have?

Problem 3. (15 points)

There are $n = 100$ people residing in Westrock Village. They are allowed to form clubs of size at least 1 and at most 5.

$a)$ What is the maximal number of clubs can be formed?

$b)$ What is the maximal number of clubs can be formed assuming no club contains another?

Problem 4. (15 points, 5 points each part)

Draw all simple graphs with given score (up to isomorphism) and prove that no other such graphs exist.

a) (2,2,2,2,1,1)

b) (2,2,2,1,1,1)

c) (5,5,5,1,1,1)

Problem 5. (30 points, 3 points each). **True or False?**

Circle the answers. Must circle in ink. No reasoning/calculations will be taken into account.

T **F** (a) The are no subgraphs of $K_{50,51}$ isomorphic to C_{10}.

T **F** (b) The are no subgraphs of $K_{50,51}$ isomorphic to C_{11}.

T **F** (c) $\binom{150}{130} > \binom{135}{115}$.

T **F** (d) An assistant wedding planner wrote in a calligraphic script the addresses on 100 envelopes and 100 personal invitations. The absentminded head wedding planner mailed out all invitations in random envelopes. Then the chance that no guest received his/her invitation is smaller than 25%.

T **F** (e) Formula $|U - (A \cup B)| = |U| - |A| - |B| + |A \cap B|$ for $A, B \subset U$ is a special case of the inclusion-exclusion principle.

T **F** (f) We have $\binom{n}{3} \sim n^3/6$ as $n \to \infty$.

T **F** (g) Graph K_5 has a unique Eulerian circuit.

T **F** (h) Graph $K_{5,10}$ has no Eulerian circuits.

T **F** (i) If a graph $G = (V, E)$ has $|V| = 20$ and $|E| = 113$, then G must contain K_3 as a subgraph.

T **F** (j) If a graph $G = (V, E)$ has $|V| = 25$ and $|E| = 76$, then G must contain $K_{2,2}$ as a subgraph.