HOMEWORK 5 (MATH 180, SPRING 2014)

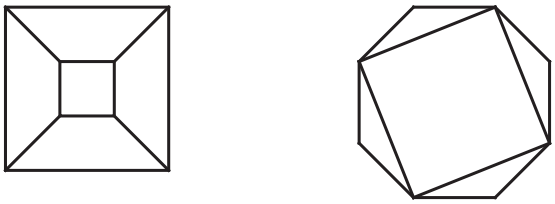
Read: **MN**, sections 8.1, 8.5, 8.6.

Solve:

I. Use determinants to compute the number of spanning trees in

a) $K_{3,4}$,
b) $K_6 - e$, where $e = (1, 2)$ is an edge in K_6,
c) $K_6 - e - e'$, where e and $e' = (3, 4)$ are edges in K_6,
d) $K_6 - e - e''$, where e and $e'' = (1, 3)$ are edges in K_6,
e) $K_7 - (1, 2) - (2, 3) - \ldots - (6, 7) - (7, 1)$.

II. Use determinants to compute the number of spanning trees in the following two graphs on 8 vertices. Which one has more?



NOTE: In I and II, you need to write your determinant calculation steps. However, before turning, consider verifying your calculations using WolframAlpha or some other computer algebra program.

III. *a*) Prove that every tree on $n \geq 2$ vertices is either a path P_n, or has at least three endpoints (end-vertices).

b) Prove that every tree on 15 vertices with one vertex of degree 5 must have at least 5 endpoints (end-vertices).

IV. A *forest* is a graph without cycles. Prove that all connected components of a forest are trees. Prove that every forest on n vertices with k connected components has exactly $n - k$ edges.

V. Suppose a connected graph G is *not* a tree. Prove that at least 3 edges of G are *not* bridges.

VI. Let G be a random subgraph of K_9 with 8 edges. Compute the probability that G is a tree.

VII. Find all trees on 7 vertices (up to isomorphism).

VIII. Use Prufer's code to compute the number of spanning trees in K_9 where the degree of vertex 1 is equal to 4.

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This Homework is due Friday May 23, at 12:59:59 pm. (right before class). Please read the collaboration policy on the course web page. Make sure you write your name in the beginning and your collaborators' names at the end. You MUST box all answers. Remember that answers are not enough, you also need to provide an explanation exhibiting your logic.

P.S. Problem I is worth 20 points; others 10 points.