HOMEWORK 2 (MATH 115A, WINTER 2010)

Read: **Friedberg**, sections 1.5, 1.6.

Solve: problems

- 2 b,d,f,h,j, 3, 8 a, 13 (the field is $\mathbb{R}$), 18 (Section 1.5)
- 2 b,d, 3 b,d, 6 (only the 2×2 matrices over $\mathbb{R}$ part), 8, 10 b,d, 11, 16 (assume that $n = 4$ here) (Section 1.6)

and the following three:

I. Let $V = \mathbb{R}^3$. *a*) Find 4 vectors such that every three of them form a basis. *b*) Find 5 vectors such that every three of them form a basis.

II. Suppose $S = \{v_1, v_2, v_3\}$ is a basis of $V = \mathbb{R}^3$. Prove that $\{-3v_2 + v_3, v_1 - v_3, v_1 + v_3\}$ is also a basis of V.

III. *Lucas numbers* are defined $L_1 = 2$, $L_2 = 1$, $L_{n+1} = L_n + L_{n-1}$ for all $n = 2, 3, ...$ Prove that $(L_{100}, L_{101}, L_{102}, \ldots)$ is a vector in the vector space of Fibonacci sequences.

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This Homework is due Wednesday January 20, at 1:59:59 pm. (right before class, no delays allowed even by 1 minute). Please read the collaboration policy on the course web page. Make sure you write your name in the beginning and your collaborators' names at the end.

You must **box** all answers. Remember that answers are not enough, you also need to prove the results, i.e. provide an explanation exhibiting your logic.

P.S. Some of these book problems are harder than others. Some are plain hard. Some have hints at the end of the book. All problems out of 10.