Upload this homework to Gradescope by Friday 6 March 2020 at 11am.

Reading:

- Chapter 14.1–2 and 16 of Newman’s book

Upload written solutions to the following problems.

Problem 1 In class we derived under which conditions networks in the configuration are locally tree-like. Use a similar argument to derive under which conditions networks in the ER model $G(n, p)$ are locally tree-like.

Problem 2 (This is Problem 14.1 in the textbook) Consider a line graph, consisting of $n$ nodes in a row like this:

\[
\bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet
\]

(i) Show that if we divide the network into two parts by cutting any single edge, such that one part has $r$ nodes and the other has $n - r$, the modularity takes the value

\[
Q = \frac{3 - 4n + 4rn - 4r^2}{2(n - 1)^2}.
\]

(ii) Hence show that the optimal division into two communities, in terms of modularity, is the division that splits the network exactly down the middle.

Problem 3 Consider the SIR model (with simple moment-closure approximation) on a network where all nodes have degree four.
(i) What is the leading eigenvalue of the adjacency matrix of the network? (Motivate your answer).

(ii) Use the value found in (i) to derive the epidemic threshold for the SIR model on this network.

(iii) In general, what information does the absolute value of the leading eigenvalue give you about the spread of a disease on a network in the SIR model? How can you interpret this in terms of the network structure?