Due: upload to Gradescope by Monday 13 January 2019 at 11am.

Reading: Chapter 6 of Newman’s book.

Problem 1. (This is exercise 6.1 in Newman’s book). Which word or words from the following list describe each of the five networks below: directed, undirected, cyclic, acyclic, approximately acyclic, planar, approximately planar, tree, approximate tree.

- The internet, at the level of autonomous systems
- A food web
- The stem and branches of a plant
- A spider web
- A complete clique of four nodes

Give one real-life example of each of the following types of networks, not including the five examples above:

- An acyclic (or approximately acyclic) directed network
- A cyclic directed network
- A tree (or approximate tree)
- A planar (or approximately planar) network
- A bipartite network

Describe briefly one empirical technique that could be used to measure the structure of each of the following networks (i.e., to fully determine the positions of all the edges):

- The World Wide Web
- A citation of scientific papers
- A food web
- A network of friendships between a group of co-workers
- A power grid
Problem 2. Let $N$ be any connected simple undirected network. What is the minimum number of edges that $N$ can have? What is the maximum number of edges that $N$ can have? Motivate your answer.

Problem 3. In class we have seen that for a simple undirected network the algebraic multiplicity of the eigenvalue 0 of the Laplacian is equal to the number of its components. For connected networks, the second smallest eigenvalue of the Laplacian is strictly greater than zero, and this eigenvalue also gives some information about the connectivity of the network. In this problem you will verify this via an example.

Consider the following network:

Compute the second smallest eigenvalue of its Laplacian. What is it?

Next, join two vertices in the two components of the network so that the network will have just one component. How does the value of the second smallest eigenvalue change, as you choose different vertices in the two components? Compute the value of the second smallest eigenvalue for three different choices of nodes.

Hint: the degree of the nodes plays a crucial role.

Problem 4. Pick a directed network of your choice and compute its number of weakly connected components and strongly connected components. For one of the strongly connected components indicate what its in- and out-components are.

Problem 5. (This is Exercise 6.6 in Newman’s book.) A “star graph” consists of a single
central node with \( n - 1 \) other nodes connected to it thus:

What is the largest (most positive) eigenvalue of the adjacency matrix of this network?

**Problem 6.** (This is Exercise 6.5(a) and (b) in Newman’s book.) Demonstrate the following for undirected networks:

- A 3-regular network must have an even number of nodes.
- The average degree of a tree is strictly less than 2.