1 Problems

Example 1.1. Two different numbers are chosen from the set \(\{1, 2, 3, \ldots, 10\}\). Find the expected value of the smaller one.

Solution

Example 1.2. You are given an unfair coin, so that \(P(\text{Head}) = \frac{1}{3}\). You throw your coin until you hit tails.

1. Compute the probability that you only roll twice.
2. Compute the probability that you roll at most four times.
3. Compute the probability that you roll at most seven times.
4. Find the expected value of number of tosses until you hit tails

Solution
Example 1.3. You are given an unfair coin, so that $P(\text{Head}) = p \in (0, 1)$ is unknown. Your goal is to approximate $p$. How can you do it?

Solution

Example 1.4. Random Variable Example Two dice are thrown: $D_1 \& D_2$. Let random variable $X$ be the sum of numbers facing up. Find $E(X)$.

Example 1.5. On average, how many rolls we need to throw a fair dice to get all 6 outcomes?

(A) 6  (B) 36  (C) 12  (D) 14.7  (E) $\frac{144}{7}$

Example 1.6. (Old MT problem) Suppose that a coin is not fair so that the probability of obtaining a head is $p \in (0, 1)$.

1. On average, how many flips are needed to obtain a head?

2. Find the probability the first time obtaining a head is an even number. Your final answer must not be an infinite series.