170E
Week 2

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1 Enumeration

Example 1.1. (PSI 1.1.1) Of a group of patients having injuries, 28% visit both a physical therapist and a chiropractor and 8% visit neither. Say that the probability of visiting a physical therapist exceeds the probability of visiting a chiropractor by 16%. What is the probability of a randomly selected person from this group visiting a physical therapist?

Solution

Problem Setup Let S be the randomly selected person. Define 2 events:

$$A = \{\text{S visits physical therapist (PT)}\}$$

and

$$B = \{\text{S visits chiropractor (Ch)}\}$$

We are given $\mathbb{P}((A \cup B)') = 0.08$, $\mathbb{P}(A \cap B) = 0.28$ and $P(A) = P(B) + 0.16$ and we are asked to find $\mathbb{P}(A)$.

Method 1 We can use *Venn Diagrams* again.

Method 2 We can use Principle of Inclusion and exclusion.

Example 1.2. (PSI 1.2.5) How many four-letter code words are possible using the letters in IOWA if

1. The letters may not be repeated?

   (A)12 (B)24 (C)48 (D)256 (E)$None$

2. The letters may be repeated?

   (A)12 (B)24 (C)48 (D)256 (E)*None*

Example 1.3. (PSI 1.2.5-modified) How many code words up to 5 letters are possible using the letters in IOWA if

1. The letters may not be repeated?

   (A)24 (B)48 (C)64 (D)256 (E)*None*

2. The letters may be repeated?

   (A)1024 (B)1228 (C)1364 (D)2048 (E)*None*

Example 1.4. A *round-robin* tournament is being held with n tennis players; this means that every player will play against every other player exactly once.

1. How many games are played in total?

2. How many possible outcomes are there for the tournament (the outcome lists out who won and who lost for each game)?

Example 1.5. A *knock-out* tournament is being held with 2^n tennis players. This means that for each round, the winners move on to the next round and the losers are eliminated, until only one person remains. For example, if initially there are $2^4 = 16$ players, then there are 8 games in the first round, then the 8 winners move on to round 2, then the 4 winners move on to round 3, then the 2 winners move on to round 4, the winner of which is declared the winner of the tournament. (There are various systems for determining who plays whom within a round, but these do not matter for this problem.)

1. How many rounds are there?

2. Count how many games in total are played, by adding up the numbers of games played in each round.

3. Count how many games in total are played, this time by directly thinking about it without doing almost any calculation.

   Hint: How many players need to be eliminated?

Example 1.6. Three people get into an empty elevator at the first floor of a building that has 10 floors. Each presses the button for their desired floor (unless one of the others has already pressed that button). Assume that they are equally likely to want to go to floors 2 through 10 (independently of each other). What is the probability that the buttons for 3 consecutive floors are pressed?

Example 1.7. Westwood High has 300 students, and it involves three active clubs: Soccer club, Basketball club and Volleyball club. Except 16 students, everybody else is engaged in at least one of the clubs. There are 130 students enrolled in Soccer Club, 100 students enrolled in Basketball club and 144 students enrolled in Volleyball club. There are 30 students who plays both soccer and basketball, 40 students who play both soccer and volleyball and 32 students who play both basketball and volleyball. Find the number of students who play all.

Example 1.8. An urn contains 8 red and 7 blue balls. A second urn contains an unknown number of red balls and 9 blue balls. A ball is drawn from each urn at random, and the probability of getting 2 balls of the same color is $\frac{151}{300}$. How many red balls are in the second urn?

Example 1.9. From the set $\{1, 2, 3, ..., n\}$, k distinct integers are selected at random and arranged in numerical order (from lowest to highest). Let $P(i, r, k, n)$ denote the probability that integer i is in position r. For example, observe that $P(1, 2, k, n) = 0$, as it is impossible for the number 1 to be in the second position after ordering. Find a general formula for $P(i, r, k, n)$.