170E
Week 1

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1 Introduction. Events as Sets

What is *Probability*: Probability is a method to quantify the possibility of occurrence of events. Let's start with an example: assume we have a class of 50 students of boys and girls, some have green eyes and the others have blue eyes.

Class		
	Boys	Girls
Blue	5	10
Green	15	20

The teacher chooses one student uniform at random in the class. Say that student is S. Let's define two events $A = \{S \text{ is a boy}\}$, $B = \{S \text{ has green eyes}\}$. The sets A and B can be represented as yellow and red colored areas

	Boys	Girls
Blue	5	10
Green	15	20
A		

	Boys	Girls
Blue	5	10
Green	15	20
B		

S is chosen uniformly random, meaning that each student in the class have the same chance to be chosen, thus

$$P(\text{S is a boy}) = p(A) = \frac{\text{Number of Boys in the Class}}{\text{Total Number of Students}} = \frac{20}{50}$$

similarly

$$p(B) = \frac{\text{Number of Green Eyed Students in the Class}}{\text{Total Number of Students}} = \frac{35}{50}$$

$$p(A \cap B) = \frac{\text{Number of Green Eyed Boys in the Class}}{\text{Total Number of Students}} = \frac{15}{50}$$

$$p(A \cup B) = \frac{\text{Number of students either are boys or have Green Eyes in the Class}}{\text{Total Number of Students}} = \frac{40}{50}$$

Note that in this example, we defined the experiment (teacher choosing one student at random), and some the event (chosen student being a male or having green eyes), then question what is the possibility/probability of these particular events are happening.

Definition 1.1. Set Operations and Their Meaning in Probability As you see in the above example, we represent events as sets. This means that we have a universal set of all possibilities, and we represent event with particular conditions as a set in the universal set. E.g., we defined $A = \{S \text{ is a boy}\},\ B = \{S \text{ has green eyes}\}$.

$$\begin{aligned}
A \cup B &= (A \text{ union } B) = (A \text{ or } B) = (S \text{ is either boy or has green eyes}) \\
A \cap B &= (A \text{ intersection } B) = (A \text{ and } B) = (S \text{ is boy and has green eyes}) \\
\mathbb{P}(A \cup B) &= \text{Probability that } A \text{ or } B \text{ happens} \\
\mathbb{P}(A \cap B) &= \text{Probability that } A \text{ and } B \text{ happens} \\
A' &= A^c = (\text{Complement of } A) = (S \text{ is not a boy}) \\
\mathbb{P}(A') &= 1 - \mathbb{P}(A)
\end{aligned}$$

Example 1.1. (PSI 1.1.7) Given that $\mathbb{P}(A \cup B) = 0.76$ and $\mathbb{P}(A \cup B') = 0.87$, find $\mathbb{P}(A)$.

Solution We can use directly the set operations. In doing so, a **Venn Diagram** is the most helpful.

Theorem 1.1 (**Principle of Inclusion & Exclusion (PIE)**). *Let A, B, C be sets in probability space $(\mathbb{P}, \Omega)$. Then*

1. $\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B)$
2. $\mathbb{P}(A \cup B \cup C) = \mathbb{P}(A) + \mathbb{P}(B) + P(C)\mathbb{P}P(A \cap B) - \mathbb{P}(A \cap C) - \mathbb{P}(B \cap C) + \mathbb{P}(A \cap B \cap C)$

Note: *It is a good example to prove above equalities*

Example 1.2. At UCLA 70 percent of students can speak either French or Spanish respectively. If half of the UCLA students can speak Spanish and 30 percent of UCLA students can speak French, what percentage of students can speak both.

Solution

Problem Setup By F and S, let's define the sets of UCLA students who can speak French and Spanish. The we are given the following:

- *In UCLA 70 percent of students can speak either French or Spanish.* $\Rightarrow \mathbb{P}(F \cup S) = 0.7$
- *Half of the UCLA students can speak Spanish* $\Rightarrow \mathbb{P}(S) = 0.5$
- *30 percent of UCLA students can speak French* $\Rightarrow \mathbb{P}(F) = 0.3$
- **Question:** *What percentage of students can speak both?* $\Rightarrow \mathbb{P}(S \cap F) =?$

Solution 1 We can use PIE. We have $\mathbb{P}(F \cup S) = \mathbb{F} + \mathbb{S} - \mathbb{P}(F \cap S)$. Hence $\mathbb{P}(F \cap S) = \mathbb{F} + \mathbb{S} - \mathbb{P}(F \cup S) = 0.3 + 0.5 - 0.7 = 0.1$, meaning 10 percent of the students can speak both of the languages.

Solution We can use directly the set operations. In doing so, a **Venn Diagram** is the most helpful.

Example 1.3. (PSI 1.1.1) Of a group of patients having injuries, 28% visit both a physical therapist and a chiropractor and 8% visit neither. Say that the probability of visiting a physical therapist exceeds the probability of visiting a chiropractor by 16%. What is the probability of a randomly selected person from this group visiting a physical therapist?

Solution

Problem Setup Let S be the randomly selected person. Define 2 events:

$$A = \{\text{S visits physical therapist (PT)}\}$$

and

$$B = \{\text{S visits chiropractor (Ch)}\}$$

We are given $\mathbb{P}((A \cup B)') = 0.08$, $\mathbb{P}(A \cap B) = 0.28$ and $P(A) = P(B) + 0.16$ and we are asked to find $\mathbb{P}(A)$.

Method 1 We can use *Venn Diagrams* again.

Method 2 We can use Principle of Inclusion and exclusion.

Example 1.4. Westwood High has 300 students, and it involves three active clubs: Soccer club, Basketball club and Volleyball club. Except 16 students, everybody else is engaged in at least one of the clubs. There are 130 students enrolled in Soccer Club, 100 students enrolled in Basketball club and 144 students enrolled in Volleyball club. There are 30 students who plays both soccer and basketball, 40 students who play both soccer and volleyball and 32 students who play both basketball and volleyball. Find the number of students who play all.