Math 170E

Week 4

Osman Akar

October 18, 2021

1 Discrete Random Variables

Example 1. Random Variable Example Two dice are thrown: D_1& D_2. Let random variable X be the sum of numbers facing up. Find

1. The pmf of X $f(2)=\frac{1}{36}, f(3)=\frac{2}{36}, \dots, f(7)=\frac{6}{36}, f(8)=\frac{5}{36}, \dots, f(12)=\frac{1}{36}$
2. $E(X)$ 7
3. $M_X(t)$, meaning moment generating function of X.

$$M_X(t)=e^{2t}\frac{1}{36}+e^{3t}\frac{2}{36}+\dots+e^{12t}\frac{1}{36}$$

$\frac{1/4}{9}$

Example 2. (PSI 2.4.1-modified) An urn contains 7 red and 11 white balls. Draw one ball at random from the urn. Let $X = 1$ if a red ball is drawn, and let $X = -1$ if a white ball is drawn. Give the pmf, mean, and variance of X.

$$f(1)=\frac{7}{18}, \quad f(-1)=\frac{11}{18}, \quad E(X)=\frac{7}{18}(1)+\frac{11}{18}(-1)=\frac{-4}{18}=\frac{-2}{9}$$

$$Var(X)=EX^2-(EX)^2=\frac{7}{18}(1)^2+\frac{11}{18}(-1)^2-\left(\frac{-2}{9}\right)^2=\frac{77}{81}$$

Example 3. (Old Quiz Problem) X is a discrete R.V. with moment generating function

$$M(t) = \frac{3e^t}{4}(1 - \frac{e^t}{4})^{-1}$$

$X = Geo(\frac{3}{4})$

1. Find the support and pmf of X $1, 2, \dots$
2. Compute the mean of X. $\frac{4}{3}$
3. Compute $Var(X)$ $\frac{4}{9}$

HINT: X is actually geometric R.V. with parameter p. Finding p is the most useful.
Remark: This problem is related to PSI-2.3.8

Example 4. **(Old Quiz Problem-PSI 2.6.10-modified)** Red Rose Tea randomly began placing one of 25 English porcelain miniature figurine in a 100-bag box of tea, selecting from ten figurine in the American Heritage series. A customer, a big fan of George Washington, wants to have three copies of George Washington figurine; one for his collection, one to give to an acquaintance, and one for his own entertainment.

On the average, how many boxes of tea must be purchased by a customer to obtain three copies of George Washington figurine?

$X = \text{NegativeBinomial}(p = \frac{1}{25}, r = 3)$

$EX = \frac{r}{p} = \frac{3}{\frac{1}{25}} = 75$

Example 5. On average, how many rolls we need to throw a fair dice to get all 6 outcomes?

(A)6 (B)36 (C)12 (D)14.7 (E)$\frac{144}{7}$

$\frac{6}{5} + \frac{6}{4} + \frac{6}{3} + \frac{6}{2} + \frac{6}{1} = 14.7$

Example 6. **(Old MT problem)** We have two fair, six-sided dice, one of which is red and the other of which is blue. We roll both dice.

1. What is the probability that at least one of the dice results in an odd number? $\frac{3}{4}$

2. What is the probability that the result of the blue die is strictly less than the result of the red die? $\frac{5}{12}$

3. Let A be the event that the sum of the rolls is even. Let B be the event that the results of both rolls are numbers ≥ 4. Are A and B independent events? Justify your answer.

No. $P(A|B) = \frac{5}{9} \neq \frac{1}{2} = P(A)$

Example 7. **(Old MT problem)** Suppose that a coin is not fair so that the probability of obtaining a head is $p \in (0, 1)$.

1. On average, how many flips are needed to obtain a head? $\frac{1}{p}$

2. Find the probability the first time obtaining a head is an even number. Your final answer must not be an infinite series.

$\sum_{k=1}^{\infty} P(X=2k) = (1-p)\cdot p + (1-p)^3 p + \cdots$

$= p(1-p)\left(1 + (1-p)^2 + (1-p)^4 + \cdots\right)$

$= p(1-p)\frac{1}{1-(1-p)^2} = \frac{p(1-p)}{2p-p^2} = \frac{1-p}{2-p}$