

Ex 4 (5.1.7) Pdf of X is equal to $f(x) = \theta \cdot x^{\theta-1}$ for $0 < x < 1$. Here, $\theta > 0$ is given value. Define $Y = -2\theta \ln X$. How Y is distributed?

A) $\exp(1)$ B) $\exp(\frac{1}{2})$ C) $\text{Unif}(0,2)$ D) $\exp(2)$

sol $x \in (0,1) \Rightarrow \ln(x) \in (-\infty, 0) \Rightarrow -2\theta \ln x \in (0, \infty)$

$$\begin{aligned}
 P(Y \leq y) &= P(-2\theta \ln X \leq y) \\
 (y \in (0, \infty)) &= P(\ln X \geq -\frac{y}{2\theta}) \\
 &= P(X \geq e^{-\frac{y}{2\theta}}) \\
 &= \int_{e^{-\frac{y}{2\theta}}}^1 f(x) dx \\
 &= \int_{e^{-\frac{y}{2\theta}}}^1 \theta \cdot x^{\theta-1} dx = \theta x^{\theta} \Big|_{x=e^{-\frac{y}{2\theta}}}^1 = 1 - (e^{-\frac{y}{2\theta}})^{\theta} \\
 &= \frac{1 - e^{-\frac{y}{2}}}{\theta} \\
 &\sim \exp(\frac{1}{2})
 \end{aligned}$$

$$F_Y(y) = 1 - e^{-\frac{y}{2}} \Rightarrow f_Y(y) = \frac{d}{dy} F_Y(y) = \frac{1}{2} e^{-\frac{y}{2}}$$

Hint: HW #4 $X \sim f(x) = \frac{\alpha x^{\alpha-1}}{\beta^{\alpha}} e^{-\left(\frac{x}{\beta}\right)^{\alpha}}$

$$M(t) = \sum_{k \geq 0} \frac{p_k t^k}{k!} \Gamma\left(1 + \frac{k}{\alpha}\right)$$

$$M(t) = E(e^{tx}) = \int e^{tx} f(x) dx$$

$$\begin{aligned}
 \hookrightarrow M(t) &= E(e^{tx}) = \int e^{tx} f(z) dz \\
 &= E\left(1 + tx + \frac{t^2 x^2}{2} + \frac{t^3 x^3}{3!} + \dots\right) \\
 &= \sum_{k \geq 0} E\left(\frac{t^k x^k}{k!}\right) = \sum_{k \geq 0} \frac{t^k}{k!} E(x^k)
 \end{aligned}$$

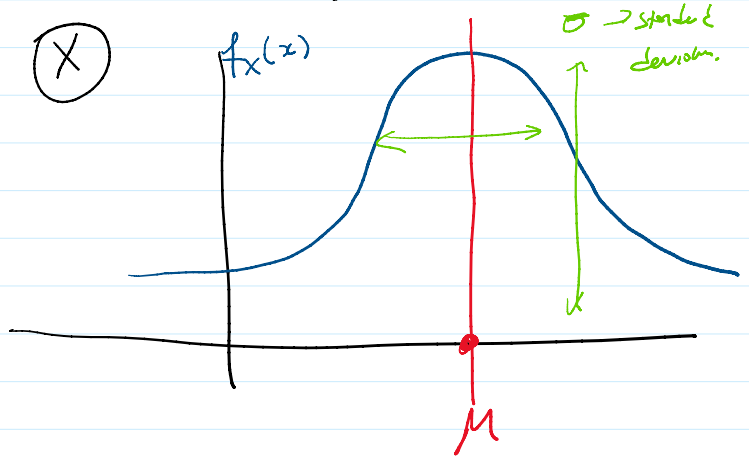
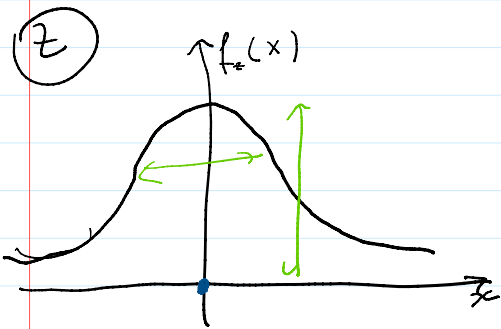
Ch 3.3 Normal RV & Standard Table

~~Ex~~ X is a normal RV with distribution $\mathcal{N}(3, 4)$. Find $P(|X| < 4)$

A) 0.13 B) 0.25 C) 0.36 D) 0.56 E) 0.69

sol $X \sim \mathcal{N}(\mu, \sigma^2)$
 μ → mean σ^2 → variance.

$$X = \mu + \sigma \underbrace{Z}_{\sim \mathcal{N}(0,1)}$$



$$X \sim \mathcal{N}(3, 4) \Rightarrow X = 3 + 2 \cdot Z \quad Z \sim \mathcal{N}(0, 1)$$

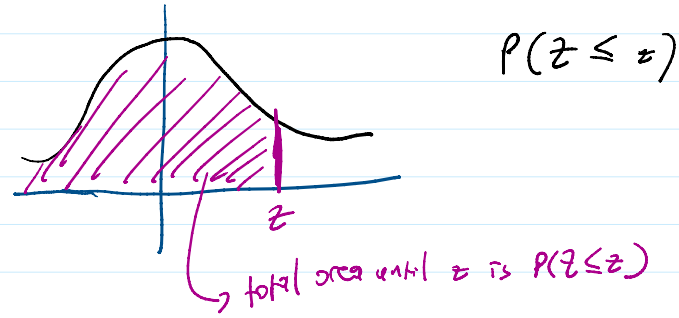
$$\begin{aligned}
 P(|X| < 4) &= P(-4 < X < 4) \\
 &= P(-4 < 3 + 2Z < 4) \\
 &= P(-7 < 2Z < 1)
 \end{aligned}$$

$$= P(-3.5 < Z < 0.5)$$



$$P(-3.5 < Z < 0.5)$$

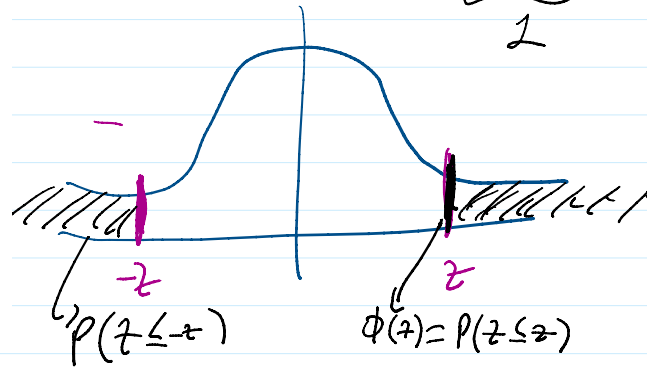
$$= P(-3.5 < z < 0.5)$$



STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5039	.5078	.5117	.5156	.5194	.5232	.5270	.5308	.5346
0.1	.5383	.5420	.5457	.5495	.5532	.5569	.5606	.5643	.5680	.5717
0.2	.5753	.5790	.5827	.5864	.5900	.5937	.5973	.6009	.6045	.6081
0.3	.6117	.6153	.6188	.6224	.6259	.6295	.6330	.6365	.6400	.6435
0.4	.6470	.6505	.6540	.6575	.6610	.6645	.6680	.6715	.6750	.6785
0.5	.6819	.6854	.6888	.6923	.6957	.6991	.7025	.7059	.7093	.7127
0.6	.7160	.7193	.7226	.7259	.7291	.7324	.7356	.7388	.7420	.7451
0.7	.7483	.7514	.7545	.7576	.7607	.7637	.7668	.7698	.7728	.7758
0.8	.7788	.7817	.7846	.7875	.7904	.7932	.7960	.7988	.8016	.8044
0.9	.8071	.8099	.8126	.8153	.8179	.8206	.8232	.8258	.8284	.8309
1.0	.8334	.8359	.8384	.8409	.8433	.8458	.8481	.8506	.8529	.8551
1.1	.8574	.8596	.8618	.8641	.8663	.8685	.8706	.8728	.8749	.8770
1.2	.8790	.8810	.8830	.8850	.8870	.8890	.8909	.8928	.8946	.8965
1.3	.8983	.8999	.9015	.9032	.9048	.9064	.9079	.9094	.9109	.9124
1.4	.9139	.9154	.9169	.9183	.9197	.9211	.9225	.9238	.9251	.9264
1.5	.9277	.9290	.9304	.9317	.9330	.9343	.9356	.9368	.9381	.9394
1.6	.9406	.9418	.9429	.9441	.9452	.9463	.9474	.9484	.9495	.9505
1.7	.9515	.9525	.9535	.9545	.9554	.9563	.9572	.9581	.9590	.9599
1.8	.9608	.9616	.9625	.9633	.9641	.9648	.9656	.9663	.9671	.9678
1.9	.9686	.9693	.9699	.9706	.9713	.9719	.9726	.9732	.9738	.9744
2.0	.9750	.9756	.9761	.9767	.9772	.9778	.9782	.9788	.9793	.9798
2.1	.9803	.9808	.9812	.9817	.9821	.9826	.9830	.9834	.9838	.9842
2.2	.9846	.9850	.9854	.9858	.9862	.9865	.9869	.9873	.9876	.9879
2.3	.9882	.9885	.9888	.9891	.9894	.9897	.9899	.9902	.9904	.9906
2.4	.9908	.9910	.9912	.9914	.9916	.9917	.9919	.9920	.9921	.9922
2.5	.9923	.9924	.9925	.9926	.9927	.9928	.9929	.9929	.9930	.9931
2.6	.9931	.9932	.9932	.9933	.9933	.9934	.9934	.9934	.9935	.9935
2.7	.9935	.9935	.9935	.9935	.9935	.9935	.9935	.9935	.9935	.9935
2.8	.9935	.9935	.9935	.9935	.9935	.9935	.9935	.9935	.9935	.9935
2.9	.9935	.9935	.9935	.9935	.9935	.9935	.9935	.9935	.9935	.9935
3.0	.9935	.9935	.9935	.9935	.9935	.9935	.9935	.9935	.9935	.9935
3.1	.9935	.9935	.9935	.9935	.9935	.9935	.9935	.9935	.9935	.9935
3.2	.9935	.9935	.9935	.9935	.9935	.9935	.9935	.9935	.9935	.9935
3.3	.9935	.9935	.9935	.9935	.9935	.9935	.9935	.9935	.9935	.9935
3.4	.9935	.9935	.9935	.9935	.9935	.9935	.9935	.9935	.9935	.9935
3.5	.9935	.9935	.9935	.9935	.9935	.9935	.9935	.9935	.9935	.9935
3.6	.9935	.9935	.9935	.9935	.9935	.9935	.9935	.9935	.9935	.9935
3.7	.9935	.9935	.9935	.9935	.9935	.9935	.9935	.9935	.9935	.9935
3.8	.9935	.9935	.9935	.9935	.9935	.9935	.9935	.9935	.9935	.9935
3.9	.9935	.9935	.9935	.9935	.9935	.9935	.9935	.9935	.9935	.9935

$$\rightarrow = \underbrace{P(z < 0.5)}_{0.69} - \underbrace{P(z < -3.5)}_{1 - \Phi(3.5)} \approx 0.69$$



$$P(z \leq -z) = 1 - P(z \leq z) = 1 - \Phi(z)$$

Q3 $X \sim N(650, 400)$ ($\mu = 650, \sigma = 20$)

(a) $P(600 \leq X \leq 660)$ A) 0.68 B) 0.72 C) 0.83 D) 0.94

(b) Find c so that $P(|X - 650| < c) = 0.95$

A) 12 B) 24.7 C) 36 D) 39.2 E) 44.3

sol $\mu = 650, \sigma^2 = 400 \Rightarrow \sigma = 20$
variance standard deviation.

$$X = 650 + 20Z \text{ where } Z \sim N(0, 1) \quad P(Z \leq z) = \Phi(z)$$

$$P(600 \leq X \leq 660) = P(600 \leq 650 + 20Z \leq 660)$$

$$= P(-50 < 20Z < 10)$$

$$1 - 0.3085 = 0.6915 = P(-50 \leq 20Z \leq 10)$$

$$= P(-50 \leq 20Z \leq 10)$$

$$= P(-2.5 \leq Z \leq 0.5)$$

$$= P(Z \leq 0.5) - P(Z \leq -2.5)$$

$$= \Phi(0.5) - \Phi(-2.5)$$

$$= \Phi(0.5) - (1 - \Phi(2.5)) =$$

STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
1.0	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96855	.96926	.96995	.97062
1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670
2.0	.97725	.97778	.97831	.97882	.97932	.97982	.98031	.98077	.98124	.98169
2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574
2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899
2.3	.98928	.98956	.98983	.99010	.99036	.99061	.99086	.99111	.99134	.99158
2.4	.99180	.99202	.99224	.99245	.99266	.99286	.99305	.99324	.99343	.99361
2.5	.99379	.99396	.99413	.99430	.99446	.99461	.99477	.99492	.99506	.99520
2.6	.99534	.99547	.99560	.99573	.99585	.99598	.99609	.99621	.99632	.99643
2.7	.99653	.99664	.99674	.99683	.99693	.99702	.99711	.99720	.99728	.99736
2.8	.99744	.99752	.99760	.99767	.99774	.99781	.99788	.99795	.99801	.99807
2.9	.99813	.99819	.99825	.99831	.99836	.99841	.99846	.99851	.99856	.99861
3.0	.99865	.99869	.99874	.99878	.99882	.99886	.99889	.99893	.99896	.99900
3.1	.99903	.99906	.99910	.99913	.99916	.99918	.99921	.99924	.99926	.99929
3.2	.99931	.99934	.99936	.99938	.99940	.99942	.99944	.99946	.99948	.99950
3.3	.99952	.99953	.99955	.99957	.99958	.99960	.99961	.99962	.99964	.99965
3.4	.99966	.99968	.99969	.99970	.99971	.99972	.99973	.99974	.99975	.99976
3.5	.99977	.99978	.99978	.99979	.99980	.99981	.99981	.99982	.99983	.99983
3.6	.99984	.99985	.99985	.99986	.99986	.99987	.99987	.99988	.99988	.99989
3.7	.99989	.99990	.99990	.99990	.99991	.99991	.99992	.99992	.99992	.99992
3.8	.99993	.99993	.99993	.99994	.99994	.99994	.99994	.99995	.99995	.99995
3.9	.99995	.99995	.99996	.99996	.99996	.99996	.99996	.99996	.99997	.99997

$$= 0.6914 - (1 - 0.9931)$$

$$= 0.68$$

(b) $P(|X - 650| \leq c) = 0.95$

$$X = 650 + 20z \rightarrow = 20z$$

$$\rightarrow P(|20z| \leq c) = P(|z| < \frac{c}{20}) =$$

$$= P(-\frac{c}{20} < z < \frac{c}{20}) = \Phi(\frac{c}{20}) - \Phi(-\frac{c}{20})$$

$$= \Phi(\frac{c}{20}) - (1 - \Phi(\frac{c}{20})) = 2\Phi(\frac{c}{20}) - 1$$

Find c so that

$$2\Phi(\frac{c}{20}) - 1 = 0.95$$

$$\Phi(\frac{c}{20}) = 0.975$$

(From table) $\frac{c}{20} = 1.96$

$$c = 20 \times 1.96 = \underline{\underline{39.2}}$$