Discussion

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(EX 5.1.3) Let X have pdf $f(x)=4x^3$ $0<x<1$. Define a new RU $Y=X^2$. Find the pdf of Y, ie $f_Y(y)$.

A) $2y$ $0<y<1$ B) $3y^2$ $0<y<1$

C) $4y^3$ $0<y<1$ D) $\frac{y}{2}$ $0<y<2$ E) 1 $0<y<1$.

sol)

$$f_Y(y) = \frac{\partial}{\partial y}\left(F_Y(y)\right)$$

X is defined on $(0,1)$, $Y=X^2 \in (0,1) \Rightarrow S_Y=(0,1)$

$$F_Y(y) = P(Y \le y) = P(X^2 \le y) \qquad (y \in (0,1))$$

$$= P(0 \le X \le \sqrt{y})$$

$$= \int_0^{\sqrt{y}} f_X(x)\,dx$$

$$= \int_0^{\sqrt{y}} 4x^3\,dx = x^4 \Big|_{x=0}^{x=\sqrt{y}} = (\sqrt{y})^4 - 0^4 = y^2$$

$$F_Y(y) = y^2 \Rightarrow f_Y(y) = \frac{\partial}{\partial y}(F_Y(y)) = \frac{\partial}{\partial y}(y^2) = 2y.$$

Ex2) Flaws in a certain type of material appear the average of one in 150 square-feet. If we assume poisson distribution, what is the prob. that at most one flaws appears in 225 square feet material?

A) 0.06 B) 0.12 C) 0.33 D) 0.56 E) 0.72

sol) X = # of flaws in 225 square feet.

$X \sim Pois(\lambda)$ $\qquad EX = \lambda$

$150 \longrightarrow 1$ $\qquad$ Rate $= \frac{1}{150}$

$225 \longrightarrow \frac{3}{2} = \lambda$

$X \sim Pois\left(\frac{3}{2}\right)$ $\qquad P(X \leq 1) = P(X=0) + P(X=1)$

$$= e^{-\frac{3}{2}} \cdot \frac{\left(\frac{3}{2}\right)^0}{0!} + e^{-\frac{3}{2}} \cdot \frac{\left(\frac{3}{2}\right)^1}{1!}$$

$$= \frac{5}{2} \cdot e^{-\frac{3}{2}} \approx 0.56$$

~~Ex3~~ (old quiz) $X \sim U([3,4])$ cts type R.V.

(a) Compute $\mu = EX$

(b) compute $MGf_X(t) = M(t)$ A) t^7 B) $t^{\frac{7t}{2}}$ C) $\frac{e^{\frac{7t}{2}}}{12}$ D) $\frac{e^{4t}-e^{3t}}{t}$ E) $\frac{e^{6t}-e^{3t}}{12t}$

(c) Compute $Var(X)$ A) 1 B) 12 C) 7 D) $\frac{1}{7}$ E) $\frac{1}{12}$

~~Sol~~ (a) $f_X(x) = 1 \quad x \in [3,4]$

$$EX = \int_3^4 x \underbrace{f_X(x)}_{1} dx = \int_3^4 x\,dx = \frac{x^2}{2}\Big|_{x=3}^{4} = \frac{4^2}{2} - \frac{3^2}{2} = \frac{7}{2}$$

(b) $M(t) = E(e^{tX}) = \int_3^4 e^{tx} \underbrace{f_X(x)}_{1} dx$

$$= \int_3^4 e^{tx} dx =$$

$$= \frac{1}{t} e^{tx}\Big|^{x=4} = \frac{1}{t} e^{4t} - \frac{1}{t} e^{3t} = \frac{e^{4t} - e^{3t}}{t}$$

$$= \frac{1}{t} e^{tx} \Big|_{x=3}^{x=4} = \frac{1}{t} e^{4t} - \frac{1}{t} e^{3t} = \frac{e^{4t} - e^{3t}}{t}$$

(c) $Var(X) = EX^2 - (EX)^2 = EX^2 - \left(\frac{7}{2}\right)^2 =$

$$EX^2 = \int_3^4 x^2 \underbrace{f_X(x)}_{1} dx = \int_3^4 x^2 dx = \frac{1}{3} x^3 \Big|_{x=3}^{4} = \frac{4^3}{3} - \frac{3^3}{3} = \frac{37}{3}$$

$$\frac{37}{3} - \frac{49}{4} = \frac{4 \times 37 - 3 \times 49}{12} = \boxed{\frac{1}{12}}$$

Ex4 (5.1.7) Pdf of X is equal to $f(x) = \theta \cdot x^{\theta - 1}$ for $0<x<1$.
Here, $\theta > 0$ is given value. Define $Y = -2\theta \ln X$. How Y is distributed?

A) $\exp(1)$ B) $\exp(\frac{1}{2})$ C) $Unif(0,2)$ D) $\exp(2)$

sol $X \in (0,1)$ $Y = \underbrace{-2\theta}\ \underbrace{\ln X}_{(-\infty,0)} \in (0,\infty) = S_Y$

$y > 0$ $F_Y(y) = P(Y \leq y)$

$= P(-2\theta \ln x \leq y)$

$= P\left(\ln x \geq \frac{-y}{2\theta}\right)$

$= P\left(X \geq e^{-\frac{y}{2\theta}}\right)$

$f_Y(y) = \frac{1}{2} e^{-\frac{y}{2}}$