Discussion

Thursday, April 15, 2021 5:20 AM

Topics: Conditional Prob & Bayesian example

Ex How many 14-letter words can we create by permuting the letters of the following

"UCLAASHECENTER"

A) $14!$ B) $\frac{14!}{2}$ C) $\frac{14!}{6}$ D) $\frac{14!}{12}$ E) $\frac{14!}{24}$

Sol $U\,C_1\,L\,A_1\,A_2\,S\,H\,E_1\,C\,E_2\,N\,T\,E_3\,R \rightarrow \frac{14!}{\underbrace{2!}_{Cs}\,\underbrace{2!}_{As}\,\underbrace{3!}_{Es}} = \frac{14!}{2\cdot 2\cdot 6} = \frac{14!}{24}$

Conditional Prob.

Ex Recall the class from the first discussion. The teacher randomly chooses one student, S, from the class.

	Boys	Girls
Blue	5	15
Green	20	10

Plays this following game

what is the eye color of S ?

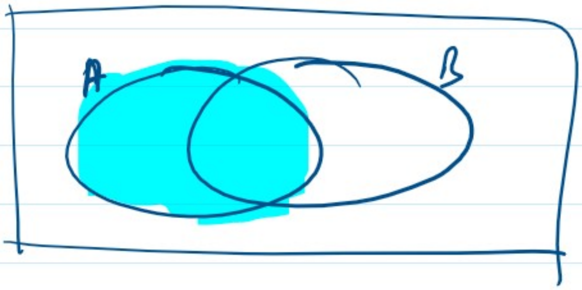
A = S has green eyes

B = S is a girl

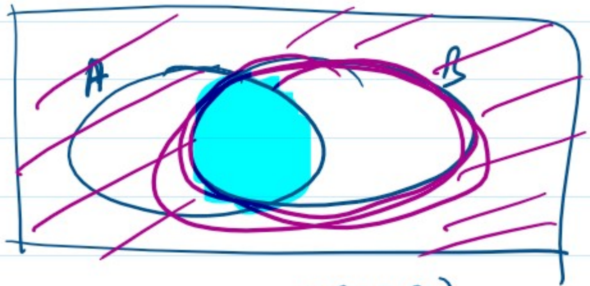
$P(A) = \frac{30}{\ } = \frac{3}{\ }$ $P(A\,|\,\underbrace{B}_{\text{Given information}}) = \frac{10}{25} = \frac{2}{\ }$

$P(A) = \frac{30}{50} = \frac{3}{5}$ $\qquad$ $P(A|B) = \frac{10}{25} = \frac{2}{5}$

P(A)



P(A|B)



$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Ex Monica throws two independent dice in a backgammon game with Rachel. You know that the sum of outcomes is 10. What is the prob. that one of the dice is 6?

A) $\frac{1}{2}$ $\quad$ B) $\frac{11}{36}$ $\quad$ C) $\frac{2}{3}$ $\quad$ D) 1 $\quad$ E) $\frac{1}{36}$

Sol $(1,6), (2,6), \ldots, (6,6), (6,5), (6,4), \ldots, (6,1) \to 11$

$A = \{D_1 = 6 \text{ or } D_2 = 6\}$ $\qquad$ $P(A) = \frac{11}{36}$

$B = \{D_1 + D_2 = 10\}$

(6,4) or (4,6)

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{2}{36}}{\frac{3}{36}} = \frac{2}{3}$$

$\hookrightarrow (4,6), (5,5), (6,4)$

$B = \{(4,6), (5,5), (6,4)\} \longrightarrow P(A|B) = \frac{2}{3}$

Ex3

Example 1.15. **(PSI 1.3.7)** An urn contains four colored balls: two orange and two blue. Two balls are selected at random without replacement, and you are told that at least one of them is orange. What is the probability that the other ball is also orange?

A) $\frac{1}{3}$ B) $\frac{1}{4}$ C) $\frac{1}{5}$ D) $\frac{1}{6}$ E) $\frac{2}{5}$

sol

$A = \{B_1 = B_2 = O\}$ $C = \{B_1 = O \text{ or } B_2 = O\}$

Q: $P(A \mid C) = ?$

$$P(A \mid C) = \frac{P(A \cap C)}{P(C)}$$

$$P(\underbrace{A \cap C}_{A \subset C}) = P(A) = P(B_1 = O) \cdot P(B_2 = O \mid B_1 = O)$$

$$= \frac{2}{4} \cdot \frac{1}{3}$$

$$= \frac{1}{6}$$

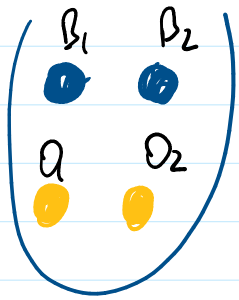
$$P(C) = P(B_1 = O \text{ or } B_2 = O) = 1 - \underbrace{P(B_1 = B_2 = \text{Blue})}_{= P(B_1 = B) \cdot P(B_2 = B \mid B_1 = B) = \frac{2}{4} \cdot \frac{1}{3} = \frac{1}{6}} = \frac{5}{6}$$

$$P(A \mid C) = \frac{P(A \cap C)}{P(C)} = \frac{\frac{1}{6}}{\frac{5}{6}} = \frac{1}{5}$$

sol2

B_1 B_2

$\cancel{\{B_1, B_2\}}$ $\{B_1, O_1\}$, $\{B_1, O_2\}$ C



$\cancel{\{B_1, B_2\}}$, $\{B_1, O_1\}$, $\{B_1, O_2\}$
$\{B_2, O_1\}$ $\{B_2, O_2\}$, $\boxed{\{O_1, O_2\}}$

Example 1.16. (PSI 1.5.9 - modified) There is a new diagnostic test for *coronavirus* that occurs in about 10% of the population. The test is not perfect, but will detect a person with the disease 95% of the time. It will, however, say that a person without the disease has the disease about 20% of the time. A person is selected at random from the population, and the test indicates that this person has the disease. What is the conditional probabilities that the person actually has the disease?

(A) $\frac{1}{3}$ (B) $\frac{9}{100}$ (C) $\frac{19}{55}$ (D) $\frac{15}{66}$ (E) $\frac{33}{95}$

$D =$ I have coronavirus. $P(D) = 0.1$

$T =$ Test says I have coronavirus. $P(T \mid D) = 0.95$