Discussion

Thursday, April 8, 2021 11:45 PM

Example 1.2. At UCLA 70 percent of students can speak either French or Spanish respectively. If half of the UCLA students can speak Spanish and 30 percent of UCLA students can speak French, what percentage of students can speak both.

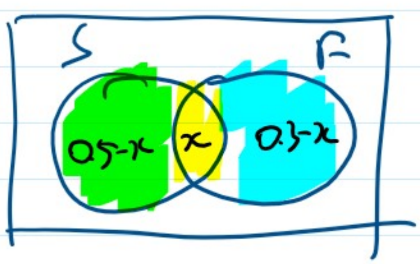
A) 5 B) 10 C) 20 D) 30 E) 40

Sol

$S = \{$ Studnts can speak Spanish $\}$

$F = \{$ Studnts can speak French $\}$

$P(S \cup F) = 0.7$ $\quad p(S) = 0.5$ $\quad P(F) = 0.3$



$Q: P(S \cap F) = x\,?$

$$P(S \cup F) = (0.5 - x) + x + (0.3 - x) = 0.7$$

$$= 0.8 - x = 0.7$$

$$x = 0.1$$

2nd Sol $\quad P(S \cup F) = P(S) + P(F) - P(S \cap F)$ $\quad$ (PIE)

$$0.7 = 0.5 + 0.3 - P(S \cap F)$$

$$P(S \cap F) = 0.1$$

EX2

Example 1.4. (PSI 1.1.1) Of a group of patients having injuries, 28% visit both a physical therapist and a chiropractor and 8% visit neither. Say that the probability of visiting a physical therapist exceeds the probability of visiting a chiropractor by 16%. What is the probability of a randomly selected person from this group visiting a physical therapist?

Modelling: $A = \{$ The selected person visits PT $\}$

Modelling: $A = \{$ The selected person visits YT $\}$

$B = \{$ " " " " Ch $\}$

$P(A \cap B) = 0.28 \qquad P((A \cup B)^c) = 0.08$

$P(A) = P(B) + 0.16 \implies P(A) = ?$

Ch 1.2 : Methods of Enumeration

Assume we have n different objects, say $O_1, O_2, \ldots, O_n$. We want to put them in an order. Then there are exactly $n! = n.(n-1)(n-2)\ldots(2).1$ ways to permute them.

$$\underset{1^{st}}{\underline{n}} \quad \underset{2^{nd}}{\underline{n-1}} \quad \underset{3^{rd}}{\underline{n-2}} \quad \underset{4^{th}}{\underline{n-3}} \quad \cdots \quad \underset{n^{th}}{\underline{1}} = n!$$

Similarly, if we want to permute r object among n different objects, $(r \le n)$ then we have

$$\underset{1^{st}}{\underline{n}} \quad \underset{2^{nd}}{\underline{n-1}} \quad \underset{3^{rd}}{\underline{n-2}} \quad \cdots \quad \underset{r^{th}}{\underline{n-r+1}} \;\Big|\; \text{will not care the rest } n-r \text{ objects.}$$

Notation $\quad {}_nP_r = n(n-1)\ldots(n-r+1)$

$$= \frac{n(n-1)\ldots(n-r+1)(n-r)(n-r-1)\ldots 2.1}{(n-r)(n-r-1)\ldots 2.1}$$

$$= \frac{n!}{(n-r)!}$$

Ex3 10 horses racing a derby. The first, the second and the third will be determined and rewarded. How many possible outcomes are there?

A) 120 B) 240 C) 360 D) 720 E) 1080

sol

$$\underset{1^{st}}{\underline{10}} \quad \underset{2^{nd}}{\underline{9}} \quad \underset{3^{r}}{\underline{8}} = 720 = {}_{10}P_3$$

Ex4 We have 10 different flavoured candies, and we want to choose 3 of them and put in a bag (all flavors must be different). How many different choices are possible?

A) 120 B) 240 C) 360 D) 720 E) 1080

sol

10	9	8

= 720 different permutations

C	L	M
C	M	L
L	C	M
L	M	C
M	C	L
M	L	C

C M L

$6 = 3!$

$\underline{3} \quad \underline{2} \quad \underline{1}$ (C, L, M)

For each combination of flavors, there are exactly $6 = 3!$ permutations.
There are in total $720 = {}_{10}P_3$ permutations, there are exactly

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$$\frac{720}{6} = \frac{10.9.8}{3.2.1} = 120 = \frac{10!}{3!\,7!}$$

different combinations

Thm: Assume we have n different object, and we want to "choose" r of them where $0 \leq r \leq n$. Then there are exactly

$${}_nC_r = C^n_r = \binom{n}{r} = \frac{{}_nP_r}{r!} = \frac{n!}{r!\,(n-r)!}$$

different combinations.

EX5 How many 8-letter words we can create by permuting the letters of

"LAGALAXY"

A) $8!$ B) $\frac{8!}{2}$ C) $\frac{8!}{4}$ D) $\frac{8!}{6}$ E) $\frac{8!}{12}$

Sol $L_1\ A_1\ G\ A_2\ L_2\ A_3\ X\ Y \longrightarrow 8!$ different permutations.

$L_2\ L_1\ A_1\ G\ A_2\ A_3\ Y\ X \longleftrightarrow$ LLAGAAYX

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In other words, I can permute A's within each other $\longrightarrow 3! = 6$ ways.

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" " " L's " " " $\longrightarrow 2! = 2$ ways.

So, there are $12 = 2 \times 6$ permutations which results in the same word.

$\Rightarrow$ There are $\frac{8!}{12}$ different words.