Osman Akk

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QF: Tue 10-11 am (right after discussion)
Friday (8-9 pm) (tentative)

Lecture notes and recordings: upload CCEL

S unless significant drop in attendance.

What is probability? Is a method to quantify the possibility of occurrence of events.

We have a class of students.

<table>
<thead>
<tr>
<th></th>
<th>Boys</th>
<th>Girls</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blue</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>Green</td>
<td>15</td>
<td>20</td>
</tr>
</tbody>
</table>

Assume the teacher chooses one student (S) uniformly random (meaning each student have chance to be chosen) from this class.

What is the prob that S has green eyes? $P(B) = ?$

What is the probability that S is a boy? $P(A) = ?$

$A = \{ S \text{ is boy} \}$

$B = \{ S \text{ has green eyes} \}$


**Problem:**

1. **Given:**
   - 15 girls have green eyes.
   - 10 boys have green eyes.
   - 30 girls do not have green eyes.
   - 20 boys do not have green eyes.

   **Question:**
   - What is the probability that a randomly selected student has green eyes?
   - What is the probability that a randomly selected student is a boy?

2. **Solution:**

   - **Probability of Green Eyes:**
     
     \[ P(\text{Green Eyes}) = \frac{15 + 10}{50} = \frac{25}{50} = \frac{1}{2} \]

   - **Probability of Boy:**
     
     \[ P(\text{Boy}) = \frac{20 + 10}{50} = \frac{30}{50} = \frac{3}{5} \]

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**Example:**

Ex: 1.17: Given that \( P(A \cup B) = 0.76 \), \( P(A') = 0.87 \), what is \( P(A) \)?

- **A) 0.24**
- **B) 0.37**
- **C) 0.50**
- **D) 0.63**
- **E) 0.77**
A) 24  B) 37  C) 50  D) 62  E) 77

\[ P((A \cup B)'^c) = 1 - 0.76 = 0.24 \]
\[ P((A \cup B)'^c) = P(A') \]
\[ = 1 - 0.87 = 0.13 \]

\[ P(A') = 0.24 + 0.13 = 0.37 \]
\[ 1 - P(A) = 0.37 \implies P(A) = 1 - 0.37 = 0.63 \]

Ex: In UCLA, 70 percent of students can speak either French or Spanish. Half of UCLA students can speak Spanish, and 30% can speak French. Find the percentage of students that can speak both.

A) 5  B) 10  C) 20  D) 40  E) 90

Sol:

Diagram showing the overlapping set of students who can speak French and Spanish.
Total region must add up to 70%.

\[
50 - x + x + 30 - x = 70
\]

\[
80 - x = 70
\]

\[
x = 10
\]

Thus, Principle of Inclusion & Exclusion (PIE)

- Let \( A \) and \( B \) be 2 arbitrary sets. Then

\[
P(A \cup B) = P(A) + P(B) - P(A \cap B)
\]

Proof:

- \( P(A) = P(A \setminus B) + P(A \cap B) \)
- \( P(B) = P(B \setminus A) + P(A \cap B) \)
- \( P(A \cup B) \) double counts \( P(A \cap B) \)

\[
P(A) + P(B) = \text{Y} + \text{E} + \text{G} + \text{P} = P(A \cup B) + P(A \cap B)
\]

\[
P(A \cup B) = P(A) + P(B) - P(A \cap B)
\]

\[
P(A \cap B)
\]
\[ P(A \cup B) = P(A) + P(B) - P(AB). \]

\[ A \setminus B = A - B, \]

represents all elements which are in A but not in B.

**PIE for 3 sets**  
A, B, C are 3 arbitrary sets, then

\[ P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(AB) - P(AC) - P(BC) + P(ABC). \]

**Ex:** In UCLA, 70% of students can speak either French or Spanish. Half of UCLA students can speak Spanish and 30% can speak French. Find the percentage of students that can speak both.

A) 5  B) 10  C) 20  D) 40  E) 90

Second Solution:  
S = \{ Students that can speak Spanish \}  
P = \{ Students that can speak French \}  

\[ P(S \cap F) = 0.7 \]
\[ P(S) = 0.5 \quad P(F) = 0.3 \quad P(S \cap F) \]
\[ P(S \cup F) = P(S) + P(F) - P(S \cap F). \]
\[ 0.7 = 0.5 + 0.3 - P(S \cap F) \]
\[ -T = 0.5 + 0.3 - \frac{P(\text{SNF})}{0.10} \]

In Example 1.11 of a group of patients having injuries, 28% visit both physical therapist (PT) and a chiropractor (Ch) and 8% visit neither. Say that prob. of visiting PT exceeds the prob. of visiting Ch. by 16%, what is the prob. of a randomly selected patient from this group visit PT?

A) 0.24  B) 0.38  C) 0.52  D) 0.68  E) 0.72