1 W1 (Ch1.1)

1.1 Logistics

1. Instructor: Dr. Kyeongsik Nam
2. Teaching Assistant: Osman Akar
3. Discussion Time: Tue 9-9.50am via Zoom
4. OH: Tue 10-11am (Right after discussion), and Fri 8-9am (Tentative). You are most welcomed to attend the office hours, even though you do not have questions but you want to see other student’s questions, or you just want to chat in general. I was also UCLA undergrad and I have spent my 4 years here, so I have quite an experience of being a math major.
5. Email: oak@math.ucla.edu.
6. Email Policy: Please do not send me math questions via email, instead come to my office hours. It is quite time consuming to answer math questions via email.
7. Lecture notes: I will upload the written notes to CCLE under the corresponding week.
8. Recorded lecture videos: I will record and upload the lecture video to CCLE, unless there is a significant drop in attendance. I do not want to lecture to an empty class, and our discussions will be interactive with question pools.
9. Useful Links:
   (a) MATLAB is free for UCLA Students: https://softwarecentral.ucla.edu/matlab-getmatlab
   (b) Microsoft Office is also free: https://www.it.ucla.edu/news/microsoft-office-proplus

1.2 Class Notes

What is Probability: Probability is a method to quantify the possibility of occurrence of events. For example, assume we have a class of 50 students of boys and girls, some have green eyes and the others have blue eyes.

<table>
<thead>
<tr>
<th>Class</th>
<th>Boys</th>
<th>Girls</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blue</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>Green</td>
<td>15</td>
<td>20</td>
</tr>
</tbody>
</table>

The teacher chooses one student uniform at random in the class. Say that student is $S$. Let’s define events $A = \{S$ is a boy$\}$, $B = \{S$ has green eyes$\}$. The sets $A$ and $B$ can be represented as yellow and red colored areas
Definition 1.1. Set Operations and Their Meaning in Probability As you see in the above example, we represent events as sets. This means that we have a universal set of all possibilities, and we represent event with particular conditions as a set in the universal set. E.g., we defined \( A = \{ \text{S is a boy} \} \), \( B = \{ \text{S has green eyes} \} \).

\[
A \cup B = (A \text{ union } B) = (A \text{ or } B) = (S \text{ is either boy or has green eyes})
\]

\[
A \cap B = (A \text{ intersection } B) = (A \text{ and } B) = (S \text{ is boy and has green eyes})
\]

\[
\mathbb{P}(A \cup B) = \text{Probability that } A \text{ or } B \text{ happens}
\]

\[
\mathbb{P}(A \cap B) = \text{Probability that } A \text{ and } B \text{ happens}
\]

\[
A' = A^c = (\text{Complement of } A) = (S \text{ is not a boy})
\]

\[
\mathbb{P}(A') = 1 - \mathbb{P}(A)
\]

Example 1.1. (PSI 1.1.7) Given that \( \mathbb{P}(A \cup B) = 0.76 \) and \( \mathbb{P}(A \cup B') = 0.87 \), find \( \mathbb{P}(A) \).

Solution We can use directly the set operations. In doing so, a Venn Diagram is the most helpful.

Theorem 1.1 (Principle of Inclusion & Exclusion (PIE)). Let \( A, B, C \) be sets in probability space \((\mathbb{P}, \Omega)\). Then

1. \( \mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B) \)
2. \( \mathbb{P}(A \cup B \cup C) = \mathbb{P}(A) + \mathbb{P}(B) + \mathbb{P}(C) - \mathbb{P}(A \cap B) - \mathbb{P}(A \cap C) - \mathbb{P}(B \cap C) + \mathbb{P}(A \cap B \cap C) \)

Example 1.2. In UCLA 70 percent of students can speak either French or Spanish. If half of the UCLA students can speak Spanish and 30 percent of UCLA students can speak French, what percentage of students can speak both.

Solution We can use directly the set operations. In doing so, a Venn Diagram is the most helpful.

Example 1.3. (PSI 1.1.1) Of a group of patients having injuries, 28% visit both a physical therapist and a chiropractor and 8% visit neither. Say that the probability of visiting a physical therapist exceeds the probability of visiting a chiropractor by 16%. What is the probability of a randomly selected person from this group visiting a physical therapist?

Solution

Problem Setup Let \( S \) be the randomly selected person. Define 2 events:

\[
A = \{ \text{S visits physical therapist (PT)} \}
\]
and

\[ B = \{ \text{S visits chiropractor (Ch)} \} \]

We are given \( P((A \cup B)') = 0.08 \) and \( P(A \cap B) = 0.28 \) and we are asked to find \( P(A) \).

**Method 1** We can use *Venn Diagrams* again.

**Method 2** We can use Principle of Inclusion and exclusion.