1 W2 (Ch1.1-1.3)

Example 1.1. (PSI 1.1.1) Of a group of patients having injuries, 28% visit both a physical therapist and a chiropractor and 8% visit neither. Say that the probability of visiting a physical therapist exceeds the probability of visiting a chiropractor by 16%. What is the probability of a randomly selected person from this group visiting a physical therapist?

Solution

Problem Setup Let S be the randomly selected person. Define 2 events:

\[ A = \{ \text{S visits physical therapist (PT)} \} \]

and

\[ B = \{ \text{S visits chiropractor (Ch)} \} \]

We are given \( \mathbb{P}(A \cup B)' = 0.08 \), \( \mathbb{P}(A \cap B) = 0.28 \) and \( P(A) = P(B) + 0.16 \) and we are asked to find \( \mathbb{P}(A) \).

Method 1 We can use Venn Diagrams again.

Method 2 We can use Principle of Inclusion and exclusion.

Theorem 1.1 (Permutation of Different Objects). Assume we have n different objects: \( O_1, O_2, \ldots, O_n \).
1. There are exactly \( n! := 1 \cdot 2 \cdot 3 \cdots (n - 1) \cdot n \) different ways to permute all of \( n \) different objects.

2. Now assume we want to permute only \( r \leq n \) of the objects. Then there exactly \( n \cdot (n - 1) \cdot (n - 1) \cdots (n - r + 1) = \frac{n!}{(n-r)!} \) ways to permute them. This is usually denoted by \( _nP_r = \frac{n!}{(n-r)!} \).

**Example 1.2.** 10 horses are racing in a derby. The first, the second, and the third will be determined and awarded. How many possible outcomes are possible?

**Solution**

**Theorem 1.2 (Combination of Different Objects).** Assume we have \( n \) different objects: \( O_1, O_2, \ldots, O_n \). Now assume we want to choose only \( r \leq n \) of the objects. Then there exactly

\[
\frac{n \cdot (n-1) \cdots (n-r+1)}{r!} = \frac{n!}{(n-r)!r!}
\]

ways to choose them. This is usually denoted by \( \binom{n}{r} = \frac{n!}{(n-r)!r!} \).

**Example 1.3.** We have 10 different flavored candies and we want to choose 3 of them. In how many ways we can do it?

**Solution**

**Example 1.4.** (PSI 1.2.5) How many four-letter code words are possible using the letters in IOWA if

1. The letters may not be repeated?

2. The letters may be repeated?

**Solution**
Example 1.5. (PSI 1.2.5-modified) How many code words up to 5 letters are possible using the letters in IOWA if

1. The letters may not be repeated?
2. The letters may be repeated?

Solution