Joint Algebra/Number theory seminar

Monday, May 22, 4.30 -6.00 pm, MS 6221

Speaker: V. Srinivas, Tata Institute

Title: Some examples of Chow groups over p-adic fields

Abstract:

After giving the relevant definitions and background on Chow groups, I'll discuss two examples constructed by Chad Schoen, over the complex number field:
(i) there exists a complex 3-fold X for which $CH^2(X) \otimes \mathbb{Z}/\ell\mathbb{Z}$ is infinite, for some primes ℓ, and
(ii) there exist complex 4-folds Y for which the ℓ-torsion subgroup of $CH^3(Y)$ is infinite.

In particular, the Griffiths group of codimension 2 cycles on X is far from being ℓ-divisible, and the ℓ-primary torsion of $CH^3(Y)$ is not a subgroup of $(\mathbb{Q}_\ell/\mathbb{Z}_\ell)^{\oplus n}$. Schoen's 4-fold examples with "big" torsion are constructed using the 3-fold example with "big" non-divisible Griffiths group of codimension 2 cycles, by taking $Y = X \times_{\mathbb{C}} E$ for a suitable elliptic curve E.

Next, I'll discuss some very recent work with Andreas Rosenschon, in which we show how to modify Schoen's arguments to get analogous examples of the following phenomena:
(i) for any odd prime p, there exists a smooth projective 3-fold X over a finite extension K_p of the field $\mathbb{Q}_p$ of p-adic numbers, such that for $\ell \in \{5, 7, 11, 13, 17\}$, the group $CH^2(X_{K_p}) \otimes \mathbb{Z}/\ell\mathbb{Z}$ is infinite, and in fact has infinite image in the Chow group (mod ℓ) over the algebraic closure of K_p (note that p may possibly equal ℓ)
(ii) for X/K_p as above, there exists a finite extension L_p of K_p and an elliptic curve E over L_p so that $CH^3(X \times_{K_p} E)$ has infinite ℓ torsion (for the same set of ℓ), and this torsion has infinite image in the Chow group over the algebraic closure of L_p.

Our modification uses the cuspidal geometry of "compactified" modular curves, in the sense of Deligne-Rapoport, and an argument with Hensel's lemma, to show that in fact Schoen's 3-fold, as well as infinitely many of his cycles, may be defined over some fixed finite extension of $\mathbb{Q}_p$, for any odd prime p.