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Title: INTERSECTIVE SETS

Abstract: Let $h(x) \in \mathbb{Z}[x]$ be a nonconstant polynomial which has a root modulo m for every positive integer m. Kamae and Mendes France have shown that given any set A of positive integers with positive upper density there exists two distinct elements $a, a' \in A$ such that

$$a - a' = h(x)$$

for some integer $x \geq 1$. Over the years several quantitative versions of this result have been given for specific polynomials. We give a quantitative result which applies to all such polynomials and from which we can deduce the following result. Given A and $h(x)$ as above we define $R(A_N)$ to be the number of solutions of

$$a - a' = h(x)$$

with $a, a' \in A \cap \{1, \ldots, N\}$ and $x \geq 1$. If the degree of $h(x)$ is $k \geq 2$, then

$$\limsup_{N \to \infty} \frac{R(A_N)}{N^{1+1/k}} > 0.$$

This generalizes a result due to R.C. Vaughan.