NUMBER THEORY SEMINAR
Monday, Feb. 2, 1.00pm- 2.00pm, MS 6943

Speaker: Samit Dasgupta , UC Berkeley

TITLE: "Elliptic Units for Real Quadratic Fields" (Joint with H. Darmon)

ABSTRACT: Through the study of elliptic units, the theory of complex multiplication allows one to understand explicit class field theory for quadratic imaginary fields. By replacing complex analysis with its p-adic counterpart, we introduce a proposal for a theory of "real multiplication."

Using the modular symbols attached to a modular unit α of level $N > 1$, we define certain numbers $u \in \mathbb{C}_p^\times$ which are intuitively analogous to the classical elliptic units arising from α. In this vein, we conjecture that the elements u belong to specific abelian extensions of real quadratic fields. Although this conjecture is still open, we are able to analyze the analytic properties of the numbers u. We prove a specific formula relating the ord_p and $\log_p$ of u to certain partial zeta functions (classical and p-adic, respectively). The second formula is analogous to the classical Kronecker Limit Formula. The existence of a unit satisfying the properties we demonstrate is the p-adic Gross-Stark conjecture; thus our construction gives an analytic construction of Gross's unit, minus a proof of its algebraicity.