UCLA Number Theory seminar: Quadratic reciprocity and sign of Gauss sum via the finite Weil representation

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I will report on a joint paper with **Ronny Hadani** (U of Chicago) and **Roger Howe** (Yale):

Two basic results due to Gauss are the quadratic reciprocity and the calculation of the sign of the Gauss sum. The first concerns the identity

$$\left(\frac{p}{q}\right)\left(\frac{q}{p}\right)=(-1)^{\frac{p-1}{2}\frac{q-1}{2}},$$

where p,q are two distinct odd prime numbers, and the latter gives an explicit evaluation of

$$G_p=\sum_{x=0}^{p-1}e^{2\pi i x^2/p}.$$

In his seminal work, Andre Weil realized the quadratic reciprocity in terms of the Weil representation of the double cover of the adele points of the group SL_2. In my talk, I will explain how these results can be understood already in the setting of the Weil representation ρ_n of the finite groups $SL_2(\mathbb{Z}/n\mathbb{Z})$ for $n=p,q$ and pq. In the course of the talk I will construct the Weil representation ρ_n and explain how it appears as a direct generalization of the discrete Fourier transform.

My talk will be elementary and **undergraduate** students and first year **graduate** students are invited to attend.