This week you will get practice solving separable differential equations, as well as some practice with linear models

*Numbers in parentheses indicate the question has been taken from the textbook:

S. J. Schreiber, *Calculus for the Life Sciences*, Wiley,

and refer to the section and question number in the textbook.

1. (6.2) Solve the following differential equations.
   (a) $\frac{dy}{dt} = 5y$
   (b) $\frac{dy}{dt} = -y$
   (c) $\frac{dy}{dx} = -3y$
   (d) $\frac{dy}{dx} = 0.2y$
   (e) (6.2-17) $\frac{dy}{dt} = y^3$
   (f) (6.2-18) $\frac{dy}{dt} = y \sin t$
   (g) (6.2-20) $\frac{dy}{dt} = \frac{t}{y}$
   (h) (6.2-24) $\frac{dy}{dx} = \frac{x}{y}\sqrt{1+x^2}$
   (i) (6.2-26) $\frac{dy}{dx} = \frac{\sin x}{\cos y}$
   (j) (6.2-30) $\frac{dy}{dt} = yt$ with $y(1) = -1$
   (k) (6.2-32) $\frac{dy}{dt} = e^{-y}t$ with $y(-2) = 0$
   (l) (6.2-34) $\frac{dy}{dt} = ty^2 + 3t^2y^2$ with $y(-1) = 2$
   (m) $\frac{dy}{dx} = y \sin x + \frac{y}{(x+1)^2}$ with $y(0) = 1$
   (n) $\frac{dy}{dx} = \frac{x}{y}e^{-x^2}$ with $y(0) = 1$
   (o) $\frac{dy}{dx} = y + ye^x$ with $y(0) = e$

2. (6.2-44) Populations may exhibit seasonal growth in response to seasonal fluctuations in resource availability. A simple model accounting for seasonal fluctuations in the abundance N of a population is

$$\frac{dN}{dt} = (R + \cos t)N$$

   where R is the average per-capita growth rate and t is measured in years.
   (a) Assume $R = 0$ and find a solution to this differential that satisfies $N(0) = N_0$. What can you say about $N(t)$ at $t \to \infty$?
   (b) Assume $R = 1$ (more generally $R > 0$) and find a solution to this differential that satisfies $N(0) = N_0$. What can you say about $N(t)$ at $t \to \infty$?
   (c) Assume $R = -1$ (more generally $R < 0$) and find a solution to this differential that satisfies $N(0) = N_0$. What can you say about $N(t)$ at $t \to \infty$?

3. (6.3-25) In 1990 the gross domestic product (GDP) of the United States was $\$5,464$ billion. Suppose the growth rate from 1989 to 1990 was 5.08%. Predict the GDP in 2003.

   (Hint: You should assume that the percentage growth rate is constant - not very realistic!)

4. (6.3-28) According to the Department of Health and Human Services, the annual growth rate in the number of marriages per year in 1990 in the United States was 9.8% and there were $2,448,000$ marriages that year. How many marriages will there be in 2004 if the annual growth rate in the number of marriages per year is constant?

5. (6.3-30) Calculate the infusion rate in milligrams per hour required to maintain a long-term drug concentration of 50 mg/L (i.e., the rate of change of drug in the body equals zero when the concentration is 50 mg/L). Assume that the half-life of the drug is 3.2 hours and that the patient has 5 L of blood.
6. (6.3-31) Calculate the infusion rate in milligrams per hour required to maintain a desired drug concentration of 2 mg/L. Assume the patient has 5.6 L of blood and the half-life of the drug is 2.7 hours.
7. (6.3-34) A drug is given at an infusion rate of 50 mg/h. The drug concentration value determined at 3 hours after the start of the infusion is 8 mg/L. Assuming the patient has 5 L of blood, estimate the half-life of this drug.
8. (6.3-37) After one hydrodynamic experiment, a tank contains 300 L of a dye solution with a dye concentration of 2 g/L. To prepare for the next experiment, the tank is to be rinsed with water flowing in at a rate of 2 L/min, with the well-stirred solution flowing out at the same rate. Write an equation that describes the amount of dye in the container. Be sure to identify variables and their units.