

This week you will get practice drawing and understanding bifurcation diagrams.

\*Numbers in parentheses indicate the question has been taken from the textbook:

S. J. Schreiber, *Calculus for the Life Sciences*, Wiley,

and refer to the section and question number in the textbook.

1. (6.6) Sketch the bifurcation diagrams for the equations in the following.

(a) (6.6-7)  $\frac{dy}{dt} = ay - y^2$

(b) (6.6-10)  $\frac{dy}{dt} = 1 - ay^2$

(c) (6.6-11)  $\frac{dy}{dt} = \sin y + a$

(d) (6.6-12)  $\frac{dy}{dt} = y^2 - ay + 2$  for  $a > 0$ .

2. (6.6) Consider the model

$$\frac{dy}{dt} = \frac{ay^2}{k^2 + y^2} - cy$$

of an autocatalytic gene from question 5. In each of the following cases, two of the parameters are specified. Sketch a bifurcation diagram with respect to the third parameter.

(a) (6.6-13)  $k = 1$ ,  $c = 2$  with  $a$  as the bifurcation parameter.

(b) (6.6-14)  $k = 2$ ,  $c = 1$  with  $a$  as the bifurcation parameter.

(c) (6.6-15)  $a = 10$ ,  $k = 1$  with  $c$  as the bifurcation parameter.

(d) (6.6-16)  $a = 10$ ,  $c = 1$  with  $k^2$  as the bifurcation parameter.

3. (6.6-40) Suppose the growth rate of a whale population at density  $N$  (individuals per million square kilometers of ocean), harvested at a rate  $h$ , is given by

$$\frac{dN}{dt} = 0.07N \left( \frac{N}{10} - 1 \right) \left( 1 - \frac{N}{80} \right) - h$$

where the units of  $t$  are years.

(a) Sketch a bifurcation diagram with respect to the parameter  $h$  as it varies over the interval  $[0, 8]$ .

(b) If  $h = vN$ , then sketch a bifurcation diagram with respect to the parameter  $v$  as it varies over the interval  $[0, 0.12]$ .