

Midterm 1

UCLA: Math 3B, Fall 2017

Instructor: Noah White

Date: 30 January 2017

- This exam has 3 questions, for a total of 36 points.
- Please print your working and answers neatly.
- Write your solutions in the space provided showing working.
- Indicate your final answer clearly.
- You may write on the reverse of a page or on the blank pages found at the back of the booklet however these will not be graded unless very clearly indicated.
- Non programmable and non graphing calculators are allowed.

Name: _____

ID number: _____

Discussion section (please circle):

Day/TA	Kevin	Bohyun	Ryan
Tuesday	3A	3C	3E
Thursday	3B	3D	3F

Question	Points	Score
1	12	
2	12	
3	12	
Total:	36	

Question 1 is multiple choice. Once you are satisfied with your solutions, indicate your answers by marking the corresponding box in the table below.

Please note! The following three pages will not be graded. You must indicate your answers here for them to be graded!

Question 1.

<i>Part</i>	A	B	C	D
(a)				
(b)				
(c)				
(d)				
(e)				

1. Each of the following questions has exactly one correct answer. Choose from the four options presented in each case. No partial points will be given.

(a) (2 points) The function $f(x) = \frac{1}{1+x^2}$ has

- A. a vertical asymptote at $x = -1$.
- B. a horizontal asymptote at $y = 0$.**
- C. no asymptotes.
- D. a slanted asymptote.

(b) (2 points) The function $g(x) = x - \sin x$ has a critical point at

- A. $x = \pi/2$.
- B. $x = 2$.
- C. $x = \pi$.
- D. $x = 0$.**

(c) (2 points) The function $f(x) = \frac{1}{5-4x+x^2}$ has a

- A. local minimum at $x = 2$.
- B. local maximum at $x = 2$.**
- C. local minimum at $x = 1$.
- D. local maximum at $x = 1$.

(d) (2 points) An antiderivative of $h(t) = 2e^{2x} - 4x$ is given by

- A. $2x^2 - \cos x^2$
- B. $2x^2 - 2e^{2x}$
- C. $e^{2x} - 2x^2 + \frac{5}{11}$
- D. $4e^{2x} - 4$

(e) (2 points) The area $\int_2^3 \ln x \, dx$ can be expressed as the limit as $n \rightarrow \infty$ of

- A. $\sum_{k=1}^n \ln \left(2 + \frac{k}{n} \right)$
- B. $\sum_{k=1}^n \frac{2}{n} + \frac{k}{n^2}$
- C. $\frac{1}{n} \sum_{k=1}^n [\ln(2n+k) - \ln n]$
- D. $\sum_{k=1}^n \frac{k}{n^2}$

(f) (2 points) Evaluate the definite integral $\int_0^\pi \cos\left(x - \frac{\pi}{2}\right) dx$

- A. 1
- B. 2
- C. π
- D. 0

2. Let $f(x) = \frac{1}{1+e^{2x}}$. Note that $f'(x) = \frac{-2e^{2x}}{(1+e^{2x})^2}$ and $f''(x) = \frac{-4e^{2x}(1-e^{2x})}{(1+e^{2x})^3}$.

(a) (2 points) Find the x and y intercepts of $f(x)$.

Solution: No x -intercepts, y -intercept at $y = 0.5$.

(b) (2 points) Does $f(x)$ have any horizontal asymptotes? If so what are they?

Solution: We need to evaluate

$$\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow \infty} \frac{1}{1+e^{2x}} = 0.$$

and

$$\lim_{x \rightarrow -\infty} f(x) = \lim_{x \rightarrow -\infty} \frac{1}{1+e^{2x}} = 1.$$

So we have horizontal asymptote in the positive direction at $y = 0$ and in the negative direction at $y = 1$.

(c) (1 point) Does $f(x)$ have any vertical asymptotes? If so what are they?

Solution: The denominator of $f(x)$ can never be zero so the function is always well defined, thus no vertical asymptotes.

(d) (2 points) For what x is the first derivative $f'(x)$ positive?

Solution: The denominator is always positive, and the numerator is always negative. Thus $f'(x)$ is negative for all x .

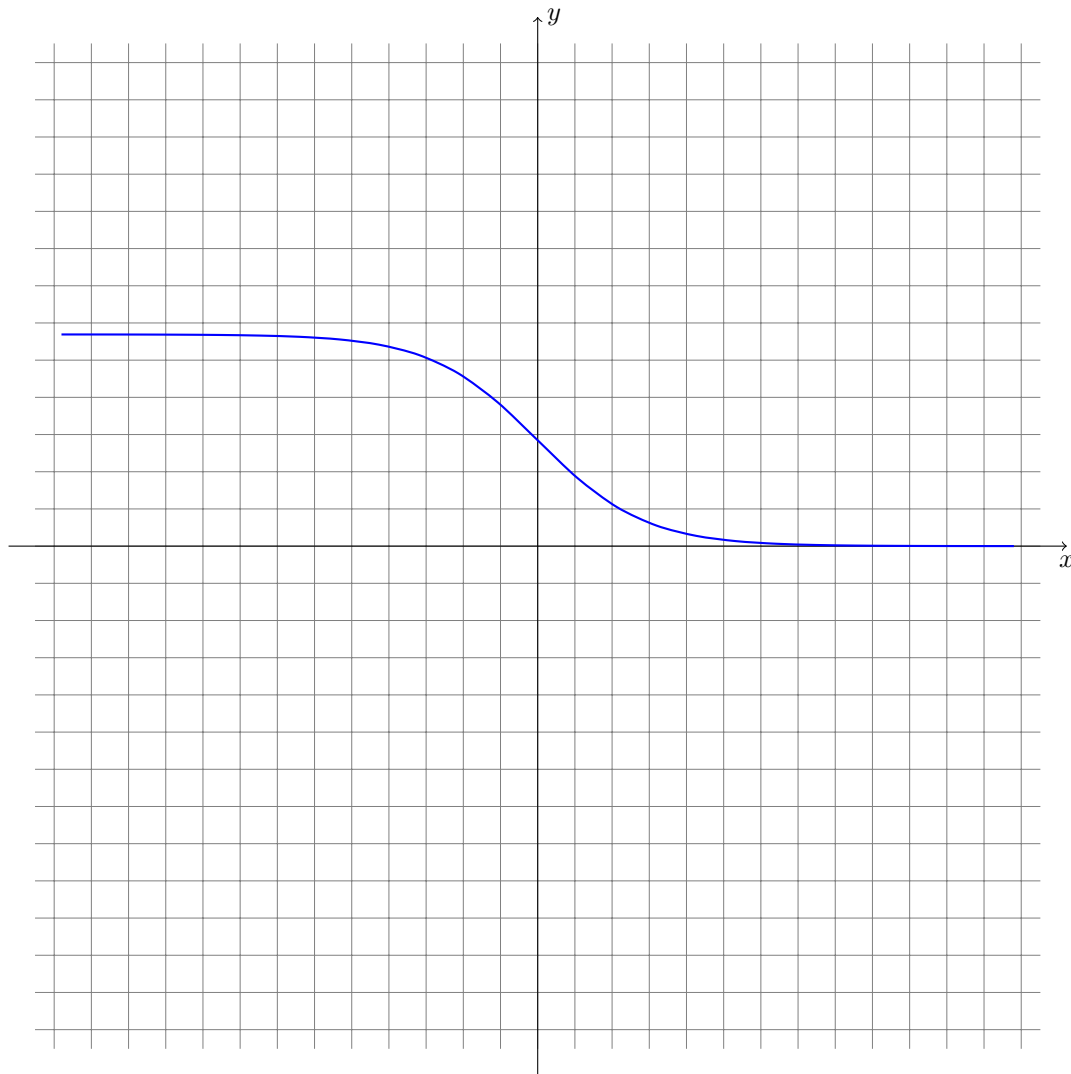
- (e) (2 points) For what x is the second derivative $f''(x)$ positive?

Solution: Again, the denominator is always positive. The numerator is positive when

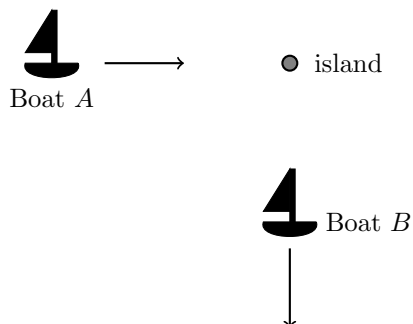
$$e^{2x} > 1$$

i.e. when $2x > 0$ thus when $x > 0$.

- (f) (3 points) On the graph provided, sketch $f(x)$



3. Two boats are travelling to and from an island in straight lines, as indicated below. Boat A is heading due east at a constant speed of 1 m/h and at time $t = 0$ is 3 miles from the island. Boat B is heading due south at 2 m/h and at time $t = 0$ is at the island. Both boats stop travelling after boat A reaches the island.



- (a) (5 points) Write down an expression for the distance $s(t)$ between the boats after t hours have elapsed.

Solution: After t hours, the distance from A to the island is $3 - t$ miles and the distance from B to the island is $2t$ miles. Thus the distance between them (using Pythagoras' theorem) is

$$d(t) = \sqrt{(3 - t)^2 + 4t^2} = \sqrt{9 - 6t + 5t^2}.$$

- (b) (2 points) What is a sensible domain for $s(t)$?

Solution: The domain for this function is $[0, 3]$ (since otherwise A reaches the island). Answers like $[0, \infty)$ were also accepted.

- (c) (5 points) At what point in time, are the boats closest together?

Solution: The derivative is

$$d'(t) = \frac{-3 + 5t}{\sqrt{9 - 6t + 5t^2}}$$

which is only zero when $t = \frac{3}{5}$. It is defined everywhere since the quadratic on the bottom never vanishes (this is easy to see since $6^2 - 4 \cdot 9 \cdot 5 < 0$, or from the physical situation since it is obvious that the boats will never be in the same place.)

Thus we only have a single critical point $t = 3/5$ on our closed interval, so using the closed interval method we just need to compare the values

$$d(0) = 3, \quad d(3/5) = \frac{6}{\sqrt{5}}, \quad d(3) = 6.$$

Since $6/\sqrt{3}$ is the smallest value (easily seen using a calculator or noticing that $\sqrt{5} > 2$ so $6/\sqrt{5} < 6/2 = 3$) we can conclude that $t = 3/5$ is the minimum (i.e. 36 minutes after starting they are closest together).

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