Math 3B: Lecture 23

Noah White

November 29, 2017

Autonomous equations

Definition

An ODE of the form

$$\frac{\mathrm{d}y}{\mathrm{d}t} = f(y)$$

i.e. where the right hand side does not depend on t, is called autonomous

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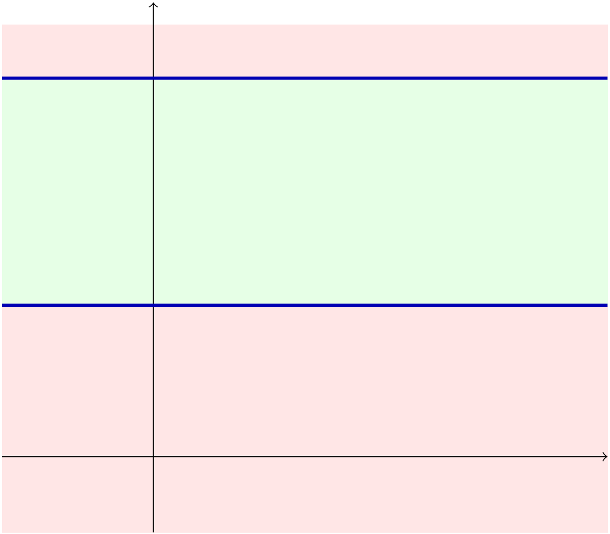
We want points (t, y) such that $f(y) = 0$.

- Suppose $f(a) = 0$.
- Then (t, a) is on the nullcline, for any t.
- So the line $y = a$ is part of the nullcline, whenever $f(a) = 0$.

Slope fields and nullclines for autonomous systems

Thus our slope field and nullclines look something like



Phase lines/diagram

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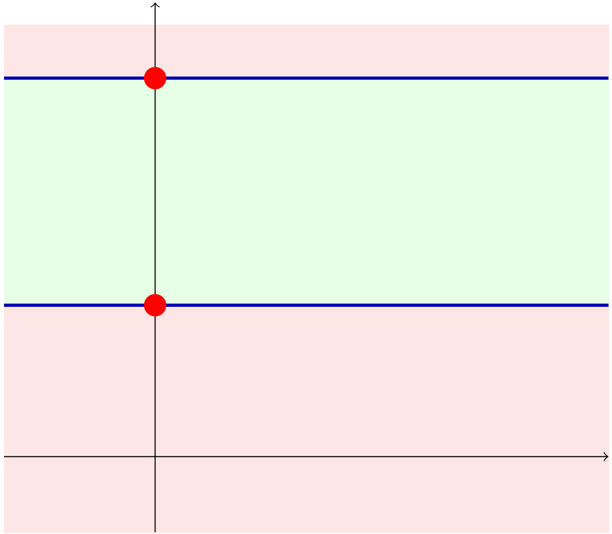

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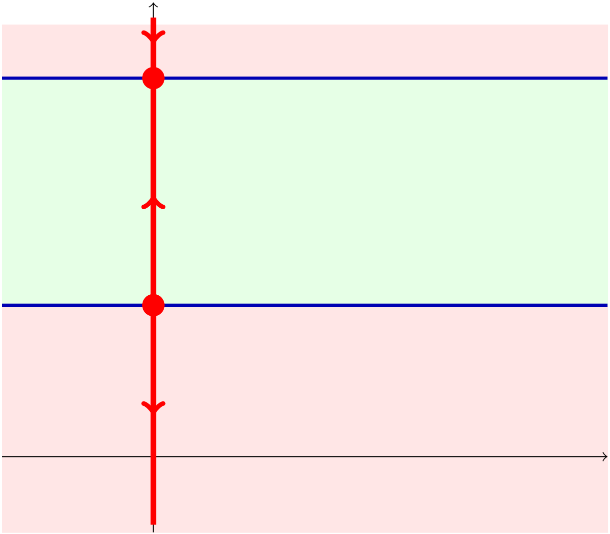

Phase lines/diagram

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Phase lines

Recipe to draw phase lines

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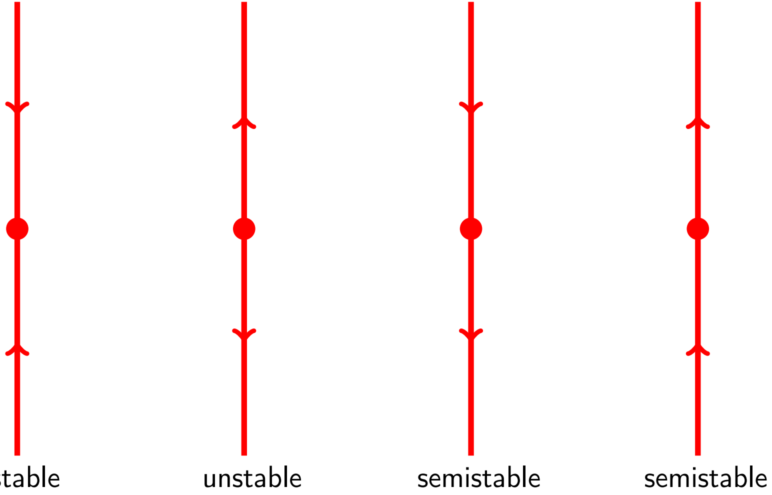
- It is unstable if the two arrows are pointing away from it.
- It is semistable if the arrows point in the same direction.

Phase lines



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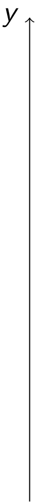

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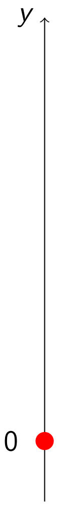

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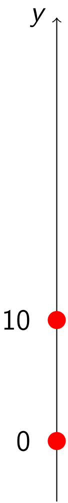

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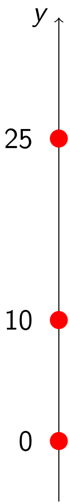

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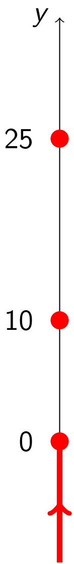

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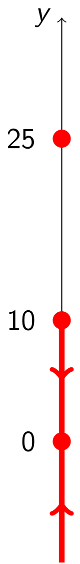

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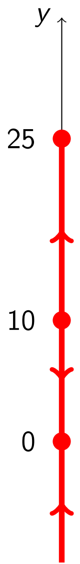

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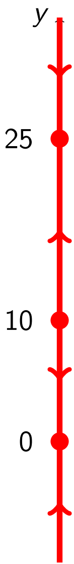

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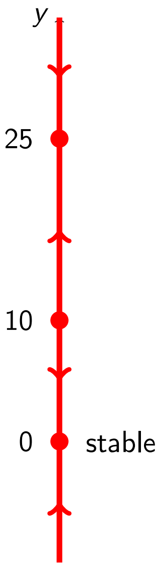

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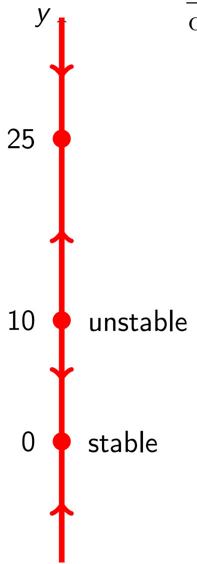

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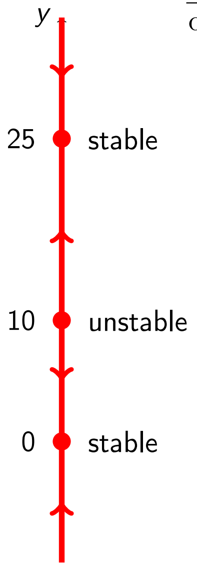

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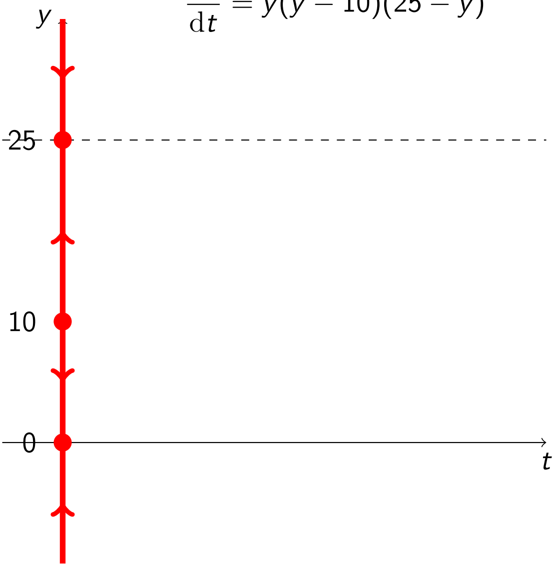

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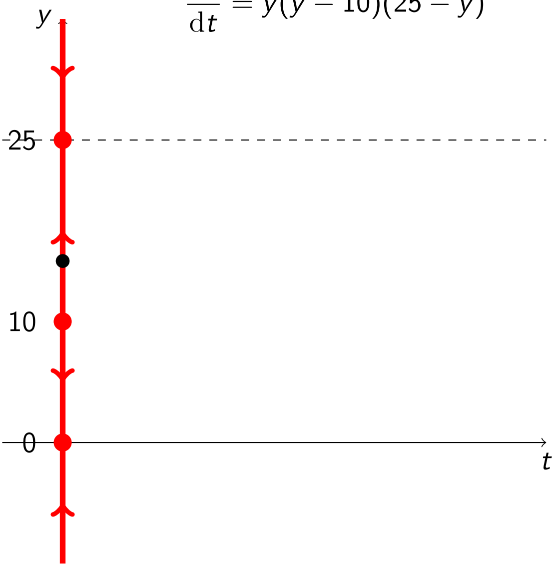

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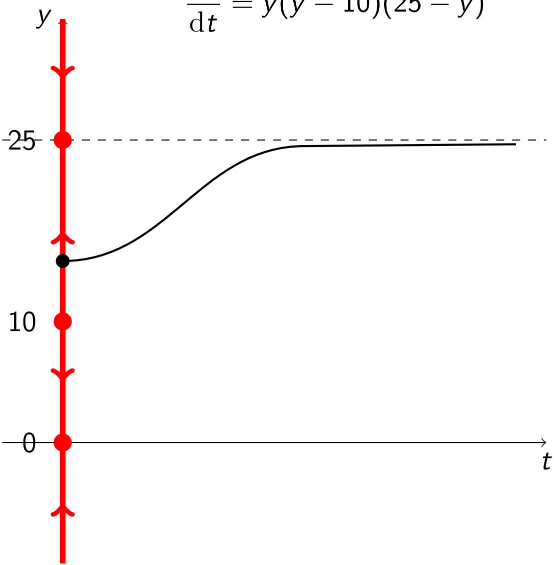

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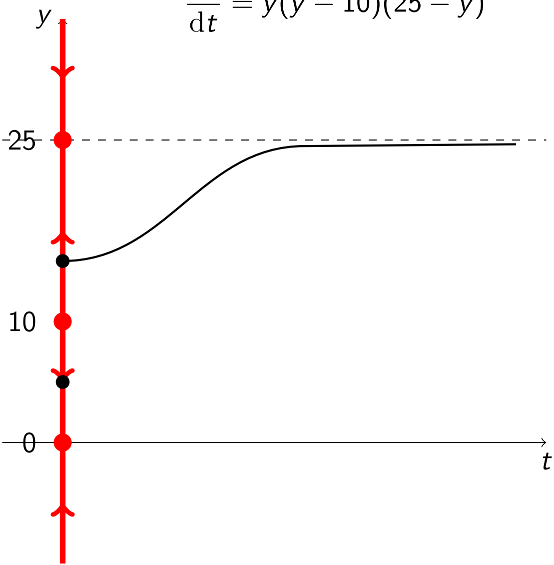

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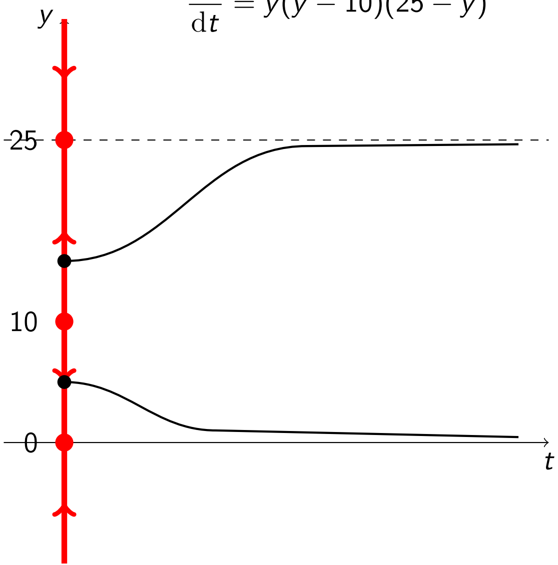

Classifying equilibria using derivatives

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- stable if $f'(a) < 0$
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- indeterminate if $f'(a) = 0$

Why?

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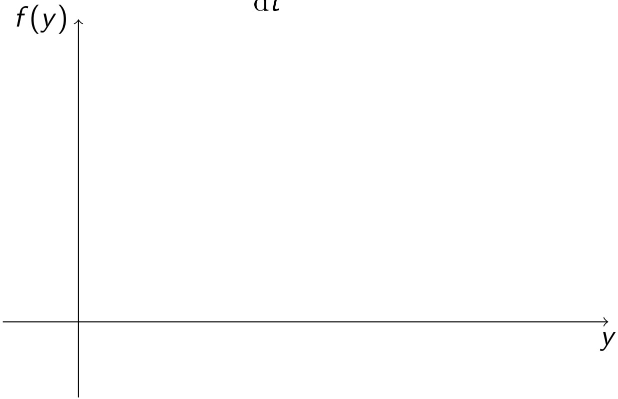

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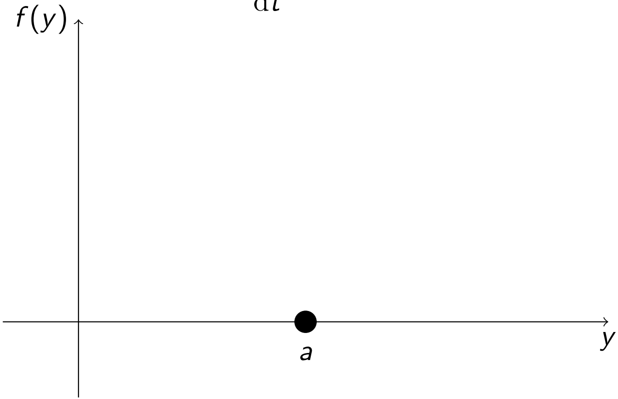

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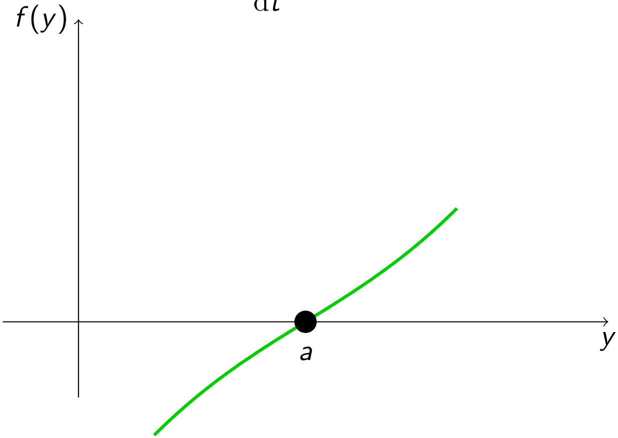

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