Math 3B: Lecture 2

Noah White

October 2, 2017

Last time

Last time, we spoke about

- The syllabus

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- Problem sets, homework, and quizzes

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- Piazza

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- Differentiation of common functions

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- Chain rule

Graphing using calculus: Why?

A common (and fair) question is: Why learn to sketch graphs when computers that do it so well?

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- Understand functions qualitatively

Graphing using calculus: Why?

A common (and fair) question is: Why learn to sketch graphs when computers that do it so well?

- Building intuition
- Understand functions qualitatively
- Better understanding of derivatives

Building intuition

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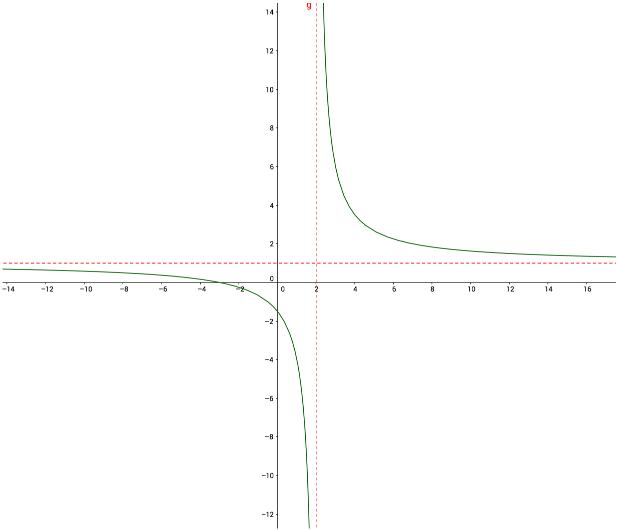
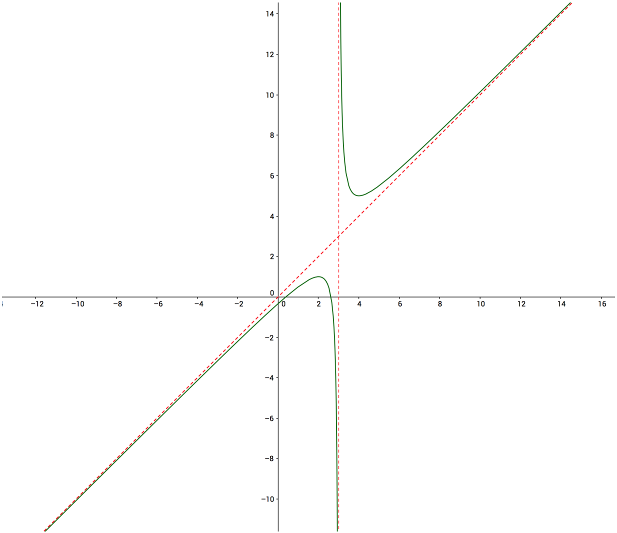
- Slanted asymptotes
- The regions of increase/decrease of the first derivative
- The regions of increase/decrease of the second derivative

Asymptotes

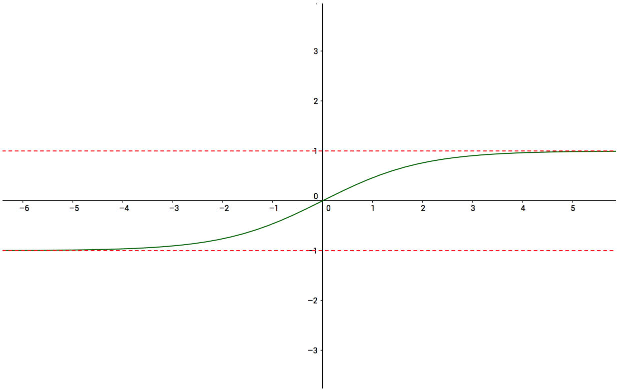
An asmptote is a line which the function approches. Some examples:



Asymptotes



Asymptotes



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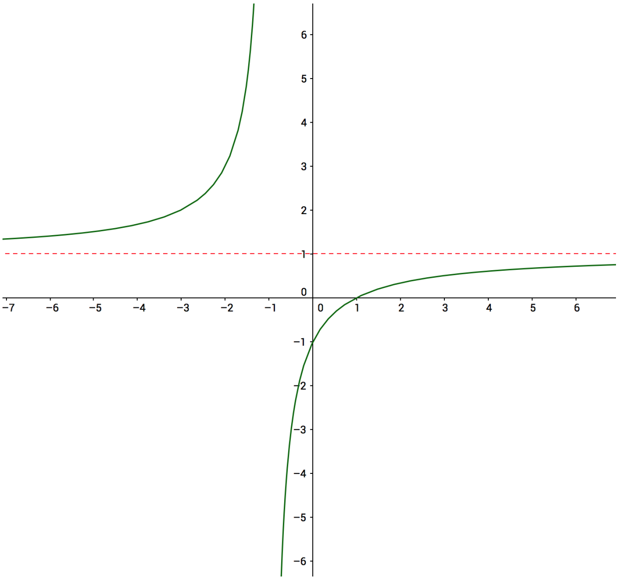
Example

Say $f(x) = \frac{x-1}{x+1}$. In this case

$$\lim_{x \to \pm\infty} \frac{x-1}{x+1} = 1$$

Finding horizontal asymptotes



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Say $f(t) = \frac{t \ln t}{t-1}$. In this case

$$\lim_{t \to \infty} \frac{t \ln t}{t-1} = \infty$$

No horizontal asymptotes.

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$$\lim_{t \to \infty} \frac{t \ln t}{t-1} = \infty$$

No horizontal asymptotes.

Example

Say $f(t) = \frac{t}{t+e^t}$. In this case

$$\lim_{t \to \infty} \frac{t}{t+e^t} = 0$$

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Finding verticle asymptotes

Verticle asymptotes happen when a function "blows up", or goes to infinity as it approches a finite number. I.e. Is there a real number a so that

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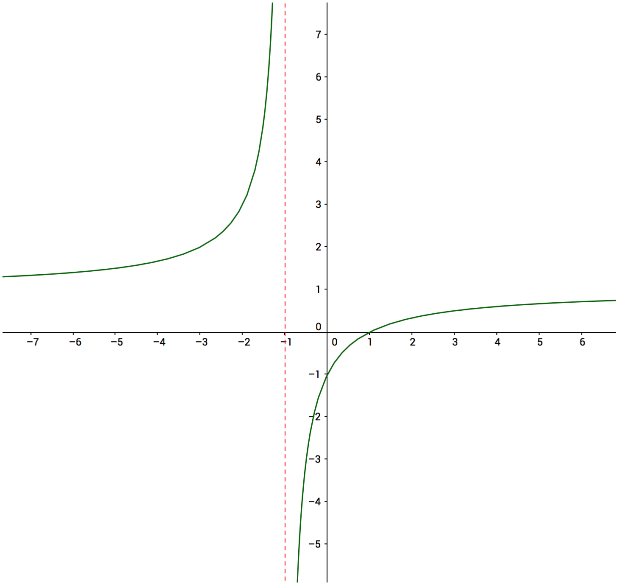
Example

$f(x) = \frac{x-1}{1+x}$, we have

$$\lim_{x \to -1^+} \frac{x-1}{1+x} = -\infty \quad \text{and} \quad \lim_{x \to -1^-} \frac{x-1}{1+x} = \infty$$

Finding verticle asymptotes

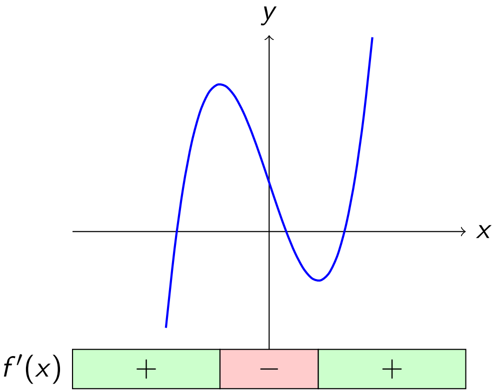


Finding slanted asymptotes

Lets come back to this. . .

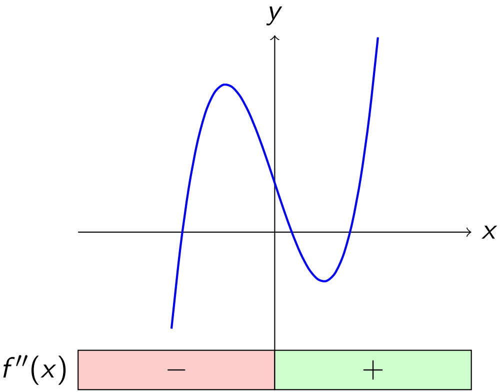
The first derivative

The first derivative tells us is the function going up or down?



The second derivative

The second derivative tells us is the function concave up or down?



Example time

. . . On the board.