Math 3B: Lecture 16

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Differential equations (motivation)

An (ordinary) differential equation (or ODE) is an equation that involves derivatives of an unknown function.

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The challenge is to find all the functions y = f(x) (or even just one) that satisfy a given equation.

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And so on.

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Note

The right hand side of the equation does not have any y's.

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- draw solutions for many other ODEs
- classify the behaviour of many ODEs (e.g. does the solution go to zero or infinity?)
- understand how sensitive ODEs are to their parameters.

Initial value problems

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we get (by integrating)

$$y(t) = t^3 + \cos t + C.$$

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- Then we see that $y(0) = 1 + C$, so $C = 1$.

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- If we want to draw the graph of $y(t)$ then we look at $g(0, 1)$.
- If this is positive we go up, negative we go down!