Math 3B: Lecture 11

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Introduction

Midterm 1

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Homework

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- Two problems to hand in Friday 3 November
- Problem 7, problem set 4

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Homework

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- Problem 7, problem set 4
- Problem 3, problem set 5

Integration by substitution

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$$\int f(g(x))g'(x)\,\mathrm{d}x = \int f(u)\,\mathrm{d}u$$

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$$\int f(g(x))\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x = \int f(g(x))g'(x)\,\mathrm{d}x = \int f(u)\,\mathrm{d}u$$

Example 1

Question

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Solution

We use the substitution $u = x^2 + 1$, so $\frac{\mathrm{d}u}{\mathrm{d}x} = 2x$, we can write the integral

$$\int 2 \cdot \sqrt{x^2+1} \cdot 2x\,\mathrm{d}x = 2\int \sqrt{u}\,\mathrm{d}u$$

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$$\begin{aligned}\int 2 \cdot \sqrt{x^2+1} \cdot 2x\,\mathrm{d}x &= 2\int \sqrt{u}\,\mathrm{d}u \\ &= 2 \cdot \frac{2}{3}u^{\frac{3}{2}} + C\end{aligned}$$

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Integration by substitution (definite integrals)

Substitution for definite integrals

Suppose $u = g(x)$, then

$$\int_a^b f(g(x))g'(x)\,\mathrm{d}x = \int_{g(a)}^{g(b)} f(u)\,\mathrm{d}u$$

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Example

$$\int_0^1 4x\sqrt{x^2+1}\,\mathrm{d}x = 2\int_1^2 \sqrt{u}\,\mathrm{d}u$$

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$$\begin{aligned}\int_0^1 4x\sqrt{x^2+1}\,\mathrm{d}x &= 2\int_1^2 \sqrt{u}\,\mathrm{d}u \\ &= 2\left[\frac{2}{3}u^{\frac{3}{2}}\right]_1^2\end{aligned}$$

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The product rule

Just like integration by substitution reverses the chain rule, integration by parts "reverses" the product rule:

$$\frac{\mathrm{d}}{\mathrm{d}x} f(x)g(x) = f'(x)g(x) + f(x)g'(x)$$

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writen another way

$$(uv)' = u'v + uv'$$

Integration by parts

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Lets integrate both sides

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By the fundamental theorem of calculus

$$uv = \int u'v \, \mathrm{d}x + \int uv' \, \mathrm{d}x$$

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Rearranging...

Integration by parts

The integration by parts formula

$$\int uv' \, \mathrm{d}x = uv - \int u'v \, \mathrm{d}x$$

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Alternative statement

$$\int u \, \mathrm{d}v = uv - \int v \, \mathrm{d}u$$

Examples

One the board. . .