This week on the problem set you will get practice applying and understanding Green’s theorem and Stokes’ theorem.

**Homework:** The homework will be due on Tuesday 3 December. It will consist of questions:

18.1.24, 18.1.36 and 18.2.19.

*Numbers in parentheses indicate the question has been taken from the textbook:

J. Rogawski, C. Adams, *Calculus, Multivariable*, 3rd Ed., W. H. Freeman & Company, and refer to the section and question number in the textbook.

1. (Section 18.1) 3, 7, 8, 9, 12, 19, 20, 21, 23, 24 25, 29, 36*, 41, 45. (Use the following translations 4th ↦ 3rd editions: 7 ↦ 5, 8 ↦ 6, 9 ↦ 7, 12 ↦ 10, 19 ↦ 15, 20 ↦ 16, 21 ↦ 17, 23 ↦ 19, 24 ↦ 20, 25 ↦ 21, 29 ↦ 25, 36 ↦ 32, 41 ↦ 37, 45 ↦ 41 otherwise the questions are the same).

2. (Section 18.2) 5, 8, 9, 18, 19. (Use the following translations 4th ↦ 3rd editions: 18 ↦ 16, 19 ↦ 17, otherwise the questions are the same).

3. (18.1.24) Find a parametrisation of the lemniscate \((x^2 + y^2)^2 = xy\) (see Figure 23) by using \(t = y/x\) as a parameter (See Exercise 23). Then use Green’s theorem to find the area of one loop of the lemniscate.

4. (18.1.36) Green’s Theorem leads to a convenient formula for the area of a polygon.

(a) Let \(C\) be the line segment joining \((x_1, y_1)\) to \((x_2, y_2)\). Show that

\[
\frac{1}{2} \int_C (-y, x) \cdot dr = \frac{1}{2}(x_1y_2 - x_2y_1).
\]

(b) Prove that the area of the polygon with vertices \((x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)\) is equal to

\[
\frac{1}{2} \sum_{i=1}^{n} (x_iy_{i+1} - x_{i+1}y_i)
\]

where \((x_{n+1}, y_{n+1}) = (x_1, y_1)\).

5. (18.2.19) Let \(I\) be the flux of \(F = \langle e^y, 2xe^x^2, z^2 \rangle\) through the upper hemisphere \(S\) of the unit sphere.

(a) Let \(G = \langle e^y, 2xe^x^2, 0 \rangle\). Find a vector field \(A\) such that \(\text{curl}(A) = G\).

(b) Use Stokes’ Theorem to show that the flux of \(G\) through \(S\) is zero. *Hint:* Calculate the circulation of \(A\) around \(\partial S\).

(c) Calculate \(I\). *Hint:* Use (b) to show that \(I\) is equal to the flux of \(\langle 0, 0, z^2 \rangle\) through \(S\).

*The questions marked with an asterisk are more difficult or are of a form that would not appear on an exam. Nonetheless they are worth thinking about as they often test understanding at a deeper conceptual level.