Midterm 2 practice 2
UCLA: Math 32B, Fall 2019

Instructor: Noah White
Date:

• This exam has 4 questions, for a total of 29 points.
• Please print your working and answers neatly.
• Write your solutions in the space provided showing working.
• Indicate your final answer clearly.
• You may write on the reverse of a page or on the blank pages found at the back of the booklet however these will not be graded unless very clearly indicated.
• Non programmable and non graphing calculators are allowed.

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ID number: 

<table>
<thead>
<tr>
<th>Question</th>
<th>Points</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9</td>
<td></td>
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<tr>
<td>2</td>
<td>8</td>
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<td>Total:</td>
<td>29</td>
<td></td>
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Here are some formulas that you may find useful as some point in the exam.

\[
\int \cos^2 x \, dx = \frac{1}{2} (x + \cos x \sin x)
\]

\[
\int \sin^2 x \, dx = \frac{1}{2} (x - \cos x \sin x)
\]

\[
\int \sin x \cos x \, dx = \frac{1}{2} \sin^2 x
\]

Spherical coordinates are given by

\[
x(\rho, \theta, \phi) = \rho \cos \theta \sin \phi
\]

\[
y(\rho, \theta, \phi) = \rho \sin \theta \sin \phi
\]

\[
z(\rho, \theta, \phi) = \rho \cos \phi
\]

The Jacobian for the change of coordinates is \( J = \rho^2 \sin \phi \).
1. Let $E$ be the solid region defined by

$$x^2 + y^2 + z^2 \leq a, \quad x, y, z \geq 0,$$

for a fixed constant $a > 0$.

(a) (2 points) Find the volume of $E$ as an iterated integral.

(b) (2 points) Find the volume of $E$. 
(c) (3 points) Let $V = \text{Vol}(E)$. Express $C_x = \frac{1}{V} \iiint_E x \, dV$, $C_y = \frac{1}{V} \iiint_E y \, dV$, and $C_z = \frac{1}{V} \iiint_E z \, dV$ as iterated integrals.

(d) (2 points) Evaluate $C_z$. 
2. Consider the helix \( \mathcal{C} \), given by the parameterisation

\[ r(t) = \left( \cos t, \sin t, \frac{1}{2\pi} t \right) \quad t \in [0, 4\pi], \]

so that \( \mathcal{C} \) is oriented with the \( z \) coordinate increasing.

(a) (4 points) Compute the length of \( \mathcal{C} \).
(b) (4 points) Compute the work done by the field

\[ \mathbf{F}(x, y, z) = (z^2, 2yz^2, 2z(x + y^2) - e^z) \]

on a particle constrained to move on the curve \( C \).
3. For this question consider the vector field

\[ \mathbf{F}(x, y) = \frac{1}{r^2} \langle y(r^2 - 1), x(r^2 + 1) \rangle, \]

where \( r = \sqrt{x^2 + y^2} \). This vector field is defined everywhere apart from the origin.

(a) (4 points) Is \( \mathbf{F} \) conservative on the domain described above? Justify your answer.

(b) (1 point) Give a domain on which \( \mathbf{F} \) is conservative.
(c) (2 points) Calculate the line integral
\[ \int_{C} \mathbf{F} \cdot d\mathbf{r} \]
where \( C \) is the ellipse \( \frac{(x-4)^2}{2} + y^2 = 1 \), oriented in the counter clockwise direction.
4. In this question assume that \( \mathbf{E} \) is a vector field defined on the whole plane, apart from the points \((\pm 1, 0)\). Suppose that \( \nabla \times \mathbf{E} = 0 \). The function \( \mathbf{r}(t) = (2 \cos t, \sin 2t) \) for \( t \in [-\frac{\pi}{2}, \frac{3\pi}{2}] \) defines the curve \( \mathcal{C} \) on the graph below.

(a) (1 point) Indicate on the above graph, the orientation of the curve.

(b) (4 points) Let \( \mathcal{A} \) and \( \mathcal{B} \) be the circles, radius \( \frac{1}{2} \), and center \((1, 0)\) and \((-1, 0)\) respectively, both oriented counter clockwise. Suppose that

\[
\int_{\mathcal{A}} \mathbf{E} \cdot d\mathbf{r} = 2 \quad \text{and} \quad \int_{\mathcal{B}} \mathbf{E} \cdot d\mathbf{r} = 1.
\]

What is \( \int_{\mathcal{C}} \mathbf{E} \cdot d\mathbf{r} \)? Justify your answer.
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