Midterm 2 practice
UCLA: Math 32B, Fall 2019

Instructor: Noah White
Date: May, 2018
Version: practice

- This exam has 4 questions, for a total of 26 points.
- Please print your working and answers neatly.
- Write your solutions in the space provided showing working.
- Indicate your final answer clearly.
- You may write on the reverse of a page or on the blank pages found at the back of the booklet however these will not be graded unless very clearly indicated.
- Non programmable and non graphing calculators are allowed.

Name: _____________________________________________________________

ID number: ________________________________________________________

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<th>Question</th>
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1. (a) (5 points) Let $D$ be the region in the $xy$-plane above the $x$-axis and below the curve $y = 1 - x^2$. Compute the integrals

$$I_1 = \frac{1}{A} \iint_D x \, dA \quad \text{and} \quad I_2 = \frac{1}{A} \iint_D y \, dA$$

where $A$ is the area of $D$. 
(b) (5 points) Find a parameterisation \( \mathbf{r}(t) \), of the curve that is the intersection of the surfaces \( y = x^2 \) and \( x + y + z = 1 \), oriented from \( x = -4 \) to \( x = 4 \), such that \( t \in [0, 1] \). What is the velocity of the parameterisation?
2. (8 points) Consider the region \( E \) given by

\[ 0 \leq z \leq (y - x^2)^2, \quad x^2 \leq y \leq x. \]

Use the change of variables

\[ x = u, \quad y = v + u^2, \quad z = uv^2, \]

to evaluate

\[ \iiint_E \frac{1}{y - x^2} \, dV. \]
3. Let $\mathbf{F}$ be the vector field on $\mathbb{R}^3$ given by

$$\mathbf{F}(x, y, z) = (y \cos z - yze^x, x \cos z - ze^x, -xy \sin z - ye^x).$$

(a) (4 points) Show that $\mathbf{F}$ is conservative.

(b) (4 points) Find a potential function for $\mathbf{F}$. 
4. Consider the vector field \( \mathbf{F} = \langle yze^{(xyz)^2}, xze^{(xyz)^2}, xyze^{(xyz)^2} + 3z^2 \rangle \). Let \( \mathbf{C} \) be the curve given by the intersection of the cylinder \( x^2 + (y - 1)^2 = 1 \) and the surface \( y = 1 - z^2 \) and \( x \geq 0 \), oriented upwards. Calculate \( \int_{\mathbf{C}} \mathbf{F} \cdot d\mathbf{r} \). You may use the fact that \( \int_{-1}^{1} e^{t^2} dt = \text{Hint: You won't be able to evaluate the integral directly. You need another method.} \)
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