This weeks problem set provides practice with diagonalisable operators and the basic properties of inner products. A question marked with a † is difficult and probably too hard for an exam (though still illustrates a useful point). A question marked with a * is especially important.

Homework 4: due Friday 2 March: questions 22a and 23 from Section 5.1.

1. From section 5.2, problems 1, 3a, d, e, 8, 9, 10, 11, 18*, 19, 20†.

2. From section 6.1, problems 1, 2, 3, 4, 8*, 9, 12, 16, 17*, 23, 29.

3.* Suppose that V is a vector space over $\mathbb{F}$ and $T : V \longrightarrow V$ is a diagonalisable map, with eigenvalues $\lambda_1, \ldots, \lambda_k$. Prove that

$$V = E_{\lambda_1} \oplus E_{\lambda_2} \oplus \cdots \oplus E_{\lambda_k}.$$

Definition: If U_i, for $1 \leq i \leq k$, are subspaces of a vector space V, then we say $V = U_1 \oplus U_2 \ldots \oplus U_k$ if $U_i \cap U_j = \{0\}$ for $i \neq j$ and $V = U_1 + U_2 + \ldots + U_k$, i.e. every vector $v \in V$ can be written as a sum $v = \sum_{i=1}^{k} u_i$ with $u \in U_i$.