MATH 131A

Analysis

Summer 2024

General information

		Lecture	Discussion
Section	Location	BOELTER 5440	PUB AFF 1337
	Time	MTR 11:00–12:50	W 11:00–12:50
Instructor	Name	Nicholas Hu (he/him)	Rushil Raghavan (he/him)
	Email	`njhu@math.ucla.edu`	`rushil@g.ucla.edu`
	Website	`math.ucla.edu/~njhu`	`rushil.online`
Office hours	Location	MS 6147	MS 6139
	Time	MTR 13:30–14:30 or by appt.	W 13:30–15:30

Acknowledgment

UCLA occupies land on the traditional, ancestral, and unceded territory of the Tongva/Kizh (Gabrieleño) people, who are the past, present, and future caretakers of Tovaangar (the Los Angeles basin and South Channel Islands).[1]

Settler colonialism continues to structure the relationship of this university and its community to the Indigenous peoples of this continent. As you take this course, I encourage you to consider the ways in which mathematics in particular has been and is still used in processes of settler colonialism and imperialism throughout the world – from the cultural supersession of Indigenous conceptions of quantity and space, to the material production of technologies for dispossession, exploitation, and elimination.[2] I also urge you to reflect on how this might inform your practice of mathematics or science beyond this course and in your future profession.

Course content

This course is an introduction to *real analysis*, the study of real numbers and functions. Real analysis is the theoretical foundation of calculus, formalizing and generalizing the concepts and results of the latter. It is also the basis of mathematical analysis as a whole, one of the major areas of modern (post-18th century) mathematics.

A detailed outline of topics is given in the Schedule section below.

Description

You have probably already encountered most of the concepts that we will study in this course – such as real numbers, sequences, series, limits, derivatives, and integrals – and you have likely performed many *calculations* related to these concepts in your *calculus* courses. For example, you probably know how to compute the limit of a

[1] For more information about local Indigenous nations, see `gabrielinotongva.org`, `gabrieleno-nsn.us`, and `gabrielenoindians.org`. For more information about Indigenous nations globally, see `native-land.ca`.

[2] An article that might provide starting points for further consideration is: A. J. Bishop. "Western mathematics: the secret weapon of cultural imperialism". *Race & Class* 32.2 (1990), pp. 51–65.

sequence or function, determine whether a series converges, and differentiate or integrate a function. But do you know, and can you precisely explain, what these concepts are and *why* these calculations work?

For instance, why is the derivative of a composition of functions given by the chain rule? Why can you compute integrals by evaluating antiderivatives and using the fundamental theorem of calculus, and is this computation always valid? What are derivatives and integrals? Do all functions have derivatives and integrals? If not, what about specific types of functions, such as continuous functions? Speaking of which, what is a continuous function, exactly? Which sequences and series converge, and how do you know? If you know from a convergence test, why does that test work?

Even "elementary" facts about real numbers may not be as "obvious" as you think they are. For example, how do you know that all nonnegative numbers have square roots? It may be apparent that 2 squared is 4, but how can you be sure that there is a number whose square is 5? Your calculator may give an approximate value of 2.236, but squaring this gives 4.999696, which is not exactly 5 – and it turns out that you will never get 5 by squaring, no matter how many digits your original number has (a fact which itself needs proof). So how can you show that a square root of 5 exists? Maybe you believe that the answer is a number with "infinitely many" digits, but this only raises more questions! How can numbers with "infinitely many" digits be multiplied (or added, or subtracted, or divided, for that matter)? Is it always possible to do so? And this leads to perhaps the most basic yet most perplexing question of them all: what even *is* a real number???

In this course, we will develop the theory to answer these questions from the ground up, starting from foundational definitions and concepts and working our way up to more complex theorems, ensuring that we understand and rigorously justify each step. This theory is what is known as *real analysis*. (As an analogy, if calculus is like *riding* a bike or *using* a computer program, then analysis is like *building* a bike or *creating* a program.) Beyond gaining deeper knowledge and understanding of these concepts, you will hopefully acquire the capability to formulate and analyze problems accurately, precisely, and thoroughly – an ability that is not only fundamental to the discipline of mathematics, but also a powerful way of reasoning about the world.

Requisites

- **MATH 32B –** Calculus of Several Variables (prerequisite)

  As real analysis is the theoretical underpinning of calculus, awareness of the notions of calculus provides useful motivation, context, intuition, and applications for those of analysis. However, MATH 131A will only develop the theory of single variable calculus; the theory of multivariable calculus will not be developed until MATH 131C, so only the calculus of MATH 31 is relevant to this course. In fact, since the theory will be developed from first principles, it is not conceptually or logically necessary to have any knowledge of calculus prior to this course.

- **MATH 33B –** Differential Equations (prerequisite)

  Differential equations will not be used in this course, despite the fact that MATH 33B is listed as a prerequisite.

- **MATH 115A –** Linear Algebra (recommended)

  Linear algebra will not be used in this course either. MATH 115A is only recommended for its use of proof techniques and more abstract mathematical reasoning, which will also be used extensively in this course.

- Mathematical proof (unofficially recommended)

  The most important requisite for this course, while not officially or explicitly stated elsewhere, is a working knowledge of mathematical proof – namely, familiarity with sets, logic, and proof techniques (e.g., direct proof, proof by contradiction, proof by contraposition, proof by induction). While these concepts will be briefly reviewed and discussed at the beginning of this course, the current department curriculum unfortunately does not allow for a detailed treatment of these topics. Consequently, you should be prepared to review or learn them implicitly through the proofs in this course or explicitly through self-study outside of class time (for recommended resources, see the Materials section).

Materials

The official course textbook is Ross's *Understanding Analysis* [Ros13]. However, I will give my own exposition and interpretation of the material, drawing on other texts such as those of Rudin [Rud76] and Strichartz [Str00]. The content of introductory real analysis courses is fairly standard and there is a plethora of texts on this subject (for instance, you might find Abbott's [Abb15] text helpful for self-study). To review or learn about mathematical proof specifically, Hammack's [Ham18] book is one possible source.

Tips for success

- Read this syllabus!
- Attend class.
- Pay attention to your instructors and classmates in class.
- Participate in the learning activities that occur in class.
- Ask questions in class and in office hours.
- Take notes in class and review them afterwards.
- Read the recommended references.
- Complete the learning assessments, especially the homework assignments.
- Collaborate and study with your classmates.
- Consider and seek feedback from your instructors and classmates, especially about errors you might have made.
- Ask for help from your instructors and classmates.
- Take care of your physical, mental, and emotional well-being!

Asking questions

Office hours are the best times to ask any course-related questions. Here are some examples of things you could ask about in office hours or elsewhere, and how you should ask.

- Homework

  If you need help with a homework problem, you should make a reasonable attempt to solve the problem beforehand and should ask *specific* questions about it. For example, rather than ask "Can you help me with this problem?", you should ask "I am trying to do A and have done B. I believe the next step is C, but I am having difficulty with this step. Can you help me with it?"

- Lectures

  As for homework, your questions should be specific. For example, rather than ask "I don't understand this proof. Can you explain it?", you should ask "I don't understand how we went from step A to step B. Can you explain this step?"

- Feedback

  You are welcome to ask for feedback on your past and present assessments. However, note that the instructors will not assess your work comprehensively before it is submitted, as this would defeat the purpose of the assessment. For instance, we will not answer questions such as "Can you check that my proof is correct?"; however, we may answer questions such as "I believe I made an error in going from step A to step B because of C, but I am not sure what it is. Can you help me find it?"

- References

  Feel free to ask about the recommended references or for additional references.

- Course policies

  Feel free to ask for clarification about any course policies.

- Accommodations and difficulties

  If you need or desire accommodations, or are experiencing difficulties in this course, feel free to bring this up in office hours if you are comfortable doing so. You can also ask such questions over email, which may be more confidential or more appropriate for your concerns.

Learning outcomes

By the end of this course, you will be able to:

- *Define* objects in real analysis (e.g., real numbers, the derivative).
- *State* theorems in real analysis (e.g., the monotone convergence theorem, the intermediate value theorem).
- *Give examples and non-examples of* objects in real analysis by verifying their definitions.
- *Apply* theorems in real analysis by verifying their hypotheses and expressing their conclusions.
- *Explain* proofs of theorems in real analysis by describing the strategies and techniques used and how relevant definitions and hypotheses are used.
- *Prove* results in real analysis by synthesizing known facts, results, and techniques.

In addition, you will be able to:

- *Evaluate* the correctness and quality of your work and your peers' work by engaging in reflection and peer review activities throughout the course.
- *Recognize the value of* writing and communication in mathematics by engaging in reflection and peer review activities throughout the course.

Assessments

The various types of assessments in this course are outlined below. Further details about these assessments, such as instructions and grading rubrics, will be provided as they arise.

Lecture notetaking

- For one lecture that you will sign up for, you will take notes that will be shared with the class (this is also known as "lecture scribing"). Your notes will be due **5 days after** the lecture at **23:59**, on Gradescope.

Peer review assignments

- A subset of the problems on each homework assignment will be designated for peer review. A *draft* solution of these problems will be due on the **following Tuesday** at **23:59** and reviews of your peers' solutions will be due on the **following Thursday** at **23:59**, both on Bruin Learn.

Homework assignments

- Homework will be assigned each Friday and will be due on the **following Friday** at **23:59**, on Gradescope.
- Collaboration with your classmates will be permitted, subject to the academic integrity policy below.

Quizzes

- There will be a two-stage quiz each **Wednesday** during part of the discussion. You will first attempt the quiz individually and then attempt the quiz in groups; only the group stage will be assessed.
- No resources will be permitted.

Midterm exam

- The midterm exam will be on **Thursday, August 22$^{\text{nd}}$** from **11:00** to **11:50**, in class.
- The only resource permitted will be a 3 in. by 5 in. index card of notes (written or typed, including both sides of the card).

Final exam

- The final exam will be on **Thursday, September 12$^{\text{th}}$** from **11:00** to **12:50**, in class.
- The only resource permitted will be a 3 in. by 5 in. index card of notes (written or typed, including both sides of the card).

Surveys

- There will be online surveys at the beginning, middle, and end of the course.
- If you complete *all* the surveys, you will be awarded bonus credit as specified below.

Grading

The contributions of the assessments to your final grade are as follows.

	Quantity	**Weight**
Lecture notetaking	1	4%
Peer reviewee assignments	5	2.5%
Peer reviewer assignments	5	2.5%
Homework assignments	*top 4 of* 5	16%
Quizzes	*top 5 of* 6	15%
Midterm exam	1	20%
Final exam	1	40%
Surveys (*bonus*)	3	1.5%

Your final percentage grade will be converted to a final letter grade according to the table below.

	F	**D–**	**D**	**D+**	**C–**	**C**	**C+**	**B–**	**B**	**B+**	**A–**	**A**	**A+**
Minimum percentage	0	60	63	67	70	73	77	80	83	87	90	93	97

I reserve the right to scale grades, but will avoid doing so.

Schedule

The following schedule is tentative and subject to change.

Week	Topics and references	Assessments
1 Aug 05 – Aug 11	Preliminaries • Logic • Sets and relations • Integers and rational numbers [Ham18, chs. 1–2, 4–12; Ros13, ss. 1–2, 8]	Quiz 1 (W)
2 Aug 12 – Aug 18	Sequences • Convergent, Cauchy, bounded, and monotone sequences • Subsequences • Limit superior and inferior [Ros13, ss. 5, 7, 9–12; Rud76, pp. 47–58] Real numbers • Construction of the real numbers • Cauchy and Dedekind completeness • Suprema and infima • Extended real numbers [Ros13, ss. 3–4; Rud76, pp. 1–12]	PR 1 due (T, R) Quiz 2 (W) HW 1 due (F)
3 Aug 19 – Aug 25	Series • Convergence tests • Absolutely and conditionally convergent series [Ros13, ss. 14–15; Rud76, pp. 58–63, 65–78]	PR 2 due (T, R) Quiz 3 (W) Midterm (R) HW 2 due (F)
4 Aug 26 – Sep 01	Point-set topology • Open and closed sets • Compact sets • Limits of functions • Continuous functions [Ros13, ss. 17–20; Rud76, pp. 30–41, 42–43, 83–94, 97–98]	PR 3 due (T, R) Quiz 4 (W) HW 3 due (F)
5 Sep 02 – Sep 08	Differentiation • The derivative • Mean value theorem • Taylor's theorem [Ros13, ss. 28–29, 31; Rud76, pp. 103–108, 110–111]	PR 4 due (T, R) Quiz 5 (W) HW 4 due (F)
6 Sep 09 – Sep 15	Integration • The integral • Fundamental theorem of calculus [Ros13, ss. 32–34; Rud76, pp. 120–135]	PR 5 due (T, R) Quiz 6 (W) Final (R) HW 5 due (F)

Policies

Contact

The best way to contact me is in person during my scheduled office hours. If you are unable to reach me then, please send me an email and include your name and UID. I am open to having office hours by appointment, but only as a last resort.

Attendance and participation

While attendance and participation in lecture and discussion are not required and will not be *directly* assessed, certain assessments (e.g., quizzes) will take place in person. In addition, I believe that the activities offered in person will enhance your learning experience and, as such, I strongly encourage you to attend and participate in them.

Late or missed assessments

Credit will not be awarded for late or missed assessments except as specified in the Grading and Accommodations sections.

Accommodations

Students needing accommodations based on a disability should first contact the Center for Accessible Education (CAE) (A255 Murphy Hall, `caeintake@saonet.ucla.edu`, 310-825-1501). All students are welcome to contact me to request unofficial or informal accommodations for reasons including (but not limited to) athletic events, emergencies, health and safety concerns, and religious observances. Any requests for accommodations should be made as early as possible to maximize the number of options available and their effectiveness, especially for those from the CAE (which should be made within the first two weeks of the quarter as per their recommendation).

Academic integrity

The regulations in Section 102.01 of the UCLA Student Conduct Code apply to this course. In particular, all work (including words and ideas) that is not your own must be appropriately attributed or cited; this includes that of non-human sources such as generative artificial intelligence. Furthermore, regardless of the sources you use, the exposition of your work must be original; it must consist of your own wording and phrasing and cannot be a copy or trivial modification of material from other sources (e.g., merely changing variable names or permuting sentences).

No specific citation style or format will be required, but enough information must be provided for others to easily locate and access the sources you used. For instance, an author, book (and edition if applicable), and page number; or a website URL; or the name and contact information of a classmate.

If you find these regulations unclear or difficult to abide by for any reason, please contact me so that I can clarify them or attempt to assist you in your situation.

Title IX

Title IX prohibits gender discrimination, including sexual harassment, domestic and dating violence, sexual assault, and stalking. If you have experienced sexual harassment or sexual violence, you can receive confidential support and advocacy at the CARE Advocacy Office for Sexual and Gender-Based Violence (330 De Neve Dr., `CAREadvocate@careprogram.ucla.edu`, 310-206-2465). In addition, Counseling and Psychological Services (CAPS) provides confidential counseling to all students and can be reached 24/7 at 310-825-0768. You can also report sexual violence or sexual harassment directly to the University's Title IX Coordinator (2241 Murphy Hall, `titleix@equity.ucla.edu`, 310-206-3417).

Resources

If you experience difficulties of any kind while taking this course, whether physical, mental, emotional, financial, or academic, you are welcome to contact me and I will try my best to help you. Please note that I am required under the UC Policy on Sexual Violence and Sexual Harassment to inform the Title IX Coordinator should I become aware that you or any other student has experienced sexual violence, sexual harassment, or other prohibited conduct.

An extensive directory of resources available to students can be found at `bewellbruin.ucla.edu`.

References

[Abb15] S. Abbott. *Understanding Analysis*. Springer New York, 2015.

[Ham18] R. Hammack. *Book of Proof*. 2018.
[Ros13] K. A. Ross. *Elementary Analysis: The Theory of Calculus*. Springer New York, 2013.
[Rud76] W. Rudin. *Principles of Mathematical Analysis*. McGraw-Hill, 1976.
[Str00] R. Strichartz. *The Way of Analysis*. Jones and Bartlett Publishers, 2000.