

# Linear algebra notation

## Vector spaces

$\text{span}(S)$	span of set $S$
$\dim(V)$	dimension of vector space $V$
$U_1 + U_2$	sum of subspaces $U_1$ and $U_2$
$U_1 \oplus U_2$	direct sum of subspaces $U_1$ and $U_2$

## Array spaces

$\mathbb{F}^n$ (over $\mathbb{F}$ )	$n$ -tuples with entries in field $\mathbb{F}$
$\mathbb{F}^{m \times n}$ (over $\mathbb{F}$ )	$m \times n$ matrices with entries in field $\mathbb{F}$

## Function spaces

$L(V, W)$ (over $\mathbb{F}$ )	linear maps from vector space $V$ to vector space $W$
$L(V)$ (over $\mathbb{F}$ )	linear maps from vector space $V$ to itself
$P(\mathbb{F})$ (over $\mathbb{F}$ )	polynomials with coefficients in field $\mathbb{F}$
$P_n(\mathbb{F})$ (over $\mathbb{F}$ )	polynomials of degree at most $n$ with coefficients in field $\mathbb{F}$
$C(X)$ (over $\mathbb{R}$ or $\mathbb{C}$ )	continuous functions on set $X$ with values in $\mathbb{R}$ or $\mathbb{C}$
$C^\infty(X)$ (over $\mathbb{R}$ or $\mathbb{C}$ )	smooth (i.e., infinitely differentiable) functions on set $X$ with values in $\mathbb{R}$ or $\mathbb{C}$

## Sequence spaces

$\mathbb{F}^{\mathbb{N}}$ (over $\mathbb{F}$ )	sequences in field $\mathbb{F}$
$\mathbb{F}^\infty$ (over $\mathbb{F}$ )	sequences in field $\mathbb{F}$ with finitely many nonzero terms

## Linear maps

$\ker(T)$	kernel of linear map $T$
$\text{im}(T)$	image of linear map $T$
$\text{null}(T)$	nullity of linear map $T$
$\text{rank}(T)$	rank of linear map $T$
$I, I_V$	identity map on vector space $V$
$T^{-1}$	inverse of linear map $T$

## Matrices

${}_B[v]$	matrix of vector $v$ with respect to basis $B$
${}_{B'}[T]_B$	matrix of linear map $T$ with respect to bases $B$ and $B'$
$a_{ij}$	$(i, j)$ -entry of matrix $A$
$A^\top$	transpose of matrix $A$
$\det(A), \det(a_1, \dots, a_n)$	determinant of matrix $A$ with columns $a_1, \dots, a_n$ (also used for linear maps)
$\text{sgn}(\sigma)$	sign of permutation $\sigma$

$e_i$	$i^{\text{th}}$ standard basis vector of $\mathbb{F}^n$
$\text{tr}(A)$	trace of matrix $A$ (also used for linear maps)

## Eigenvalues and eigenvectors

$E_T(\lambda)$	eigenspace of eigenvalue $\lambda$ of linear map $T$
$\gamma_T(\lambda)$	geometric multiplicity of eigenvalue $\lambda$ of linear map $T$
$\alpha_T(\lambda)$	algebraic multiplicity of eigenvalue $\lambda$ of linear map $T$
$\chi_T$	characteristic polynomial of linear map $T$
$\mu_T$	minimal polynomial of linear map $T$

## Normed and inner product spaces

$\ v\ $	norm of vector $v$
$\langle v, w \rangle$	inner product of vectors $v$ and $w$
$\text{proj}_w(v), P_w(v)$	orthogonal projection of vector $v$ onto span of vector $w$
$\text{proj}_W(v), P_W(v)$	orthogonal projection of vector $v$ onto subspace $W$
$S^\perp$	orthogonal complement of set $S$
$T^*$	adjoint of linear map $T$
$A^*$	conjugate transpose of matrix $A$

## Complex numbers

$\bar{z}$	complex conjugate of complex number $z$
$ z $	absolute value of complex number $z$
$\Re(z)$	real part of complex number $z$
$\Im(z)$	imaginary part of complex number $z$

## Norms and inner products

$\langle v, w \rangle_2$	standard inner/dot product of $v, w \in \mathbb{F}^n$ (where $\mathbb{F} \in \{\mathbb{R}, \mathbb{C}\}$ ) $:= \sum_{i=1}^n v_i \overline{w_i}$
$\ v\ _2$	standard/2- norm of $v, w \in \mathbb{F}^n$ (where $\mathbb{F} \in \{\mathbb{R}, \mathbb{C}\}$ ) $:= \langle v, v \rangle_2^{1/2}$