Math 115A: Week 7 Notes

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Eigenvalues and Eigenvectors Continued 1

MONDAY LECTURE

Let V be an n-dimensional vector space over a field \mathbb{F} -vector space, and $T \in L(V)$.

Remark 1.1. We sometimes call linear maps "linear operators".

Definition 1.2. If λ is an eigenvalue of T, its

Geometric Multiplicity $\gamma_T(\lambda)$ is $\dim(\underbrace{\ker(\lambda I - T)}_{\text{Eigenspace}E_T(\lambda)}) = \operatorname{null}(\lambda I - T).$

 $1 \le \gamma_T(\lambda) \le n$

Algebraic Multiplicity $\alpha_T(\lambda)$ is

the multiplicity of λ as a root of

 $\underbrace{\chi_T(\lambda) \coloneqq \det(\lambda I - T)}_{\text{Characteristic Polynomial}}$

 $1 \le \alpha_T(\lambda) \le n$

Proposition 1.3. $\gamma_T(\lambda) \leq \alpha_T(\lambda)$: the geometric multiplicity of an eigenvalue λ is at most the algebraic multiplicity of λ

Suppose λ is an eigenvalue of T and $\gamma_T(\lambda) = m$. Then, there exist v_1, \ldots, v_m such that $\{v_1, \ldots, v_m\}$ is a basis of $E_T(\lambda)$. Extend this to a basis $B := \{v_1, \ldots, v_m, v_{m+1}, \ldots, v_n\}$ of V.

Question 1.4. Questions to think about

- 1. What is/what form does ${}_{B}[T]_{B}$ have?
- 2. What about the characteristic polynomial of this matrix?

Example 1.5.

$$T(x_1, x_2) = (3x_1 - x_2, x_1 + x_2) \longrightarrow$$
 Eigenvalues: $\lambda = 2, \alpha_T(2) = 2$
 $m = \gamma_T(2) = 1$

1.

Г												
	λ	0	0	0	0	0	?	?	?	?	?	?
	0	λ	0	0	0	0	?	?	?	?	?	?
	0	0	·	0	0	0	?	?	?	?	?	?
	0	0	0	·.	0	0	?	?	?	?	?	?
	0	0	0	0	λ	0	?	?	?	?	?	?
	0	0	0	0	0	λ	?	?	?	?	?	?
	0	0	0	0	0	0	?	?	?	?	?	?
	0	0	0	0	0	0	?	?	?	?	?	?
	0	0	0	0	0	0	?	?	?	?	?	?
	0	0	0	0	0	0	?	?	?	?	?	?
	0	0	0	0	0	0	?	?	?	?	?	?
	0	0	0	0	0	0	?	?	?	?	?	?
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 ${}_B[Tv_1]_B \cdots {}_B[Tv_m]_{B B}[Tv_{m+1}]_B \cdots {}_B[Tv_n]_B$

Back Table: Try different extensions:

 $\begin{cases} \begin{bmatrix} 1\\1 \end{bmatrix}, \begin{bmatrix} 1\\0 \end{bmatrix} \} \quad \left\{ \begin{bmatrix} 1\\1 \end{bmatrix}, \begin{bmatrix} 0\\1 \end{bmatrix} \right\} \quad \left\{ \begin{bmatrix} 1\\1 \end{bmatrix}, \begin{bmatrix} -1\\0 \end{bmatrix} \right\} \quad \left\{ \begin{bmatrix} 1\\1 \end{bmatrix}, \begin{bmatrix} 2\\1 \end{bmatrix} \right\}$ $B[T]_B = \begin{bmatrix} 2 & ?\\0 & 2 \end{bmatrix}$

In each trial,

Gary Observed: By the way, ${\rm det}_{B}[T]_{B}=[T]_{original\ basis}$

2.

 $det(xI - {}_B[T]_B) = (x - \lambda)^m \cdot ?$ x = λ is necessarily a solution. *multiplicity of m

2 Eigenvalues and Eigenvectors Continued

TUESDAY DISCUSSION

V is finite-dimensional. Take λ eigenvlaue of T, v_1, \dots, v_m basis of ker(λ I-T). Extend to basis B= $v_1, \dots, v_m, v_{m+1}, \dots, v_m$ of V.

Question 2.1. If we know ${}_{B}[T]_{B}$, then what can we say about $\chi_{T}(x)$?

Claim 2.2. $(x-\lambda)$ appears at least m times in $\chi_T(x)$.

———— Observe that:

$x - \lambda$	0	0	0	?	?	?	?
0	۰.	0	0	?	?	?	?
0	0	۰.	0	?	?	?	?
0	0	0	$x - \lambda$?	?	?	?
0	0	0	0	?	?	?	?
0	0	0	0	?	?	?	?
0	0	0	0	?	?	?	?
0	0	0	0	?	?	?	?

Evaluate det(xI - T) using the permutation expansion:

ways to choose distinct rows from each column
$$\longrightarrow \sum_{\sigma} \operatorname{sgn}(\sigma) a_{\sigma(1),1} \cdot a_{\sigma(2),2} \cdots a_{\sigma(n),n}$$

For each permutation σ , if $\sigma(1) \neq 1$, then the corresponding term $\operatorname{sgn}(\sigma) \ a_{\sigma(1),1} \cdots \ a_{\sigma(n),n} = 0$ because $a_{\sigma(1),1} = 0$. So, only the terms with $\sigma(1) = 1$ survive. In fact, by the same reasoning, all terms with $\sigma(2) \neq 2 \cdots \sigma(n) \neq n$ vanishes. So, we know all surviving terms have $\sigma(1) = 1 \cdots \sigma(m) = m$. So, each of these terms shares the factors $a_{11} \ a_{22} \cdots a_{mm} = (x - \lambda)^m$ We just showed $(x - \lambda)^m$ is a factor in each term of $\det(\lambda I - T) = \Sigma$ So $(x - \lambda)^m$ is a factor of $\det(\lambda I - T)$. Conclusion:

$$\underbrace{\alpha_T(\lambda)}_{\text{algebraic multiplicity}} \ge m = \underbrace{\gamma_T(w)}_{\text{geometric multiplicity}}$$

adriday: Induction / Quiz neview Let V be a vector space our $F = T \in L(U)$ claim: for any n'and for any v,..., v, if v,..., v, are eigenvictors of T with distinct eigenvalues then v,..., va are linearly independent. This peop for N=2 was coursed yesterday. Today, we did a genuel proof by induction using n=2 as the base case: Using base case from turblay: Proof by Tame k. all Inductive hypothesis: Suppose $\sum_{i=1}^{k} \alpha_i v_i = 0$ iff. $\alpha_i = 0$ where v, ... vn are eigen notors for dottined eigenvaluers. Now Inppose q, v, + ... + q v k ta er v kr, = 0 Û $T(\alpha, \nu, \tau \dots \tau \sigma_{k}\nu_{k} \tau \alpha_{k\tau}, \nu_{k\tau}) = \alpha, \tau \nu_{\tau} \tau \dots \tau \tau_{k} \tau \alpha_{k\tau}, \overline{\nu_{k\tau}}, \overline{\nu_{k\tau}$ Now take (1) × 7 K+1 $\alpha_1 \lambda_{k+1} V_1 + \dots + \alpha_k \lambda_{k+1} V_k + \alpha_{k+1} \lambda_{k+1} V = 0$

Justant () Brown (2): $\alpha_{i}v_{i}\left(\lambda_{i}-\lambda_{k+i}\right)+\dots+\alpha_{k}v_{k}\left(\lambda_{k}-\lambda_{k+i}\right)+\alpha_{i}\left(\lambda_{i}-\lambda_{k+i}\right)=0$ By inductive hypothesis, because v, ... v, are C.I. and 2, and 121 are distivet and v; is an agenvitor, $\alpha_i v_i (\lambda_i - \lambda_{k+i}) = 0$ a: must equal zero -> a:,..., a = 0. clusert back into (1): $\begin{array}{c} \alpha_{k+1} V_{k+1} = 0 \\ \text{Since } V_{k+1} \neq 0, \ \alpha_{k+1} = 0. \\ \text{Thus, } V_{1, \dots} \quad V_{k+1} \text{ is liverly independent.} \\ \end{array}$

3 Eigenvalues and Eigenvectors Continued

THURSDAY DISCUSSION

Diagonalization

Let V be an n-dimensional vector space over a field \mathbb{F} -vector space, and $T \in L(V)$.

Definition 3.1. T is DIAGONALIZABLE if there exists a basis B of V such that $_B[T]_B$ is diagonal.

Let $a_{ij} \in \mathbb{F}$. Then the diagonal matrix is given by:

$\begin{bmatrix} a_{11} \\ 0 \end{bmatrix}$	$0 \\ a_{22}$	$\begin{array}{c} 0 \\ 0 \end{array}$	0 - 0				
0	0	·	0	0	0	0	0
0	0	0	·	0	0	0	0
0	0	0	0	·	0	0	0
0	0	0	0	0	·	0	0
0	0	0	0	0	0	·	0
$\begin{bmatrix} 0 \end{bmatrix}$	0	0	0	0	0	0	a_{nn}

Theorem 3.2. If $\lambda_1, ..., \lambda_m$ are the distinct eigenvalues of T (these numbers are different from each other), then the following are equivalent:

(a) T is diagonalizable (b) V has a basis fo eigenvectors of T

(c)
$$V = \bigoplus_{i=1}^{m} E_T(\lambda_i)$$

Direct Sum: For all $v \in V$, there exists unique $v_i \in E_T(\lambda_i)$ such that

$$v = \sum_{i=1}^{m} v_i$$

(d)
$$n = \sum_{i=1}^{m} \gamma_T(\lambda_i)$$

(e) χ_T has n zeroes (counted with multiplicity), and $\gamma_T(\lambda_i) = \alpha_T(\lambda_i)$ for each *i*.

Counted Multiplicity:

$$\sum_{i=1}^{m} \alpha_i(\lambda_i) = n$$

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Diagonalization	• •	• •	•	• •	• •	• •	
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Let V be a n-dim. F -vector space. and T $\in L(v)$.	· ·	• •		· ·	· ·	· ·	· · ·
Theorem If $\lambda_1, \ldots, \lambda_m$ are the distinct eigenvalues of T,	Hhen	the	follow	ing an	e equi	valent:	
(a) T is diagonalizable	• •				• •	• •	
if any 1 is (b) V has a basis of eigenvectors of T					• •	• •	
all are true (c) $V = \bigoplus E_{\tau}(\lambda_i)$			•	• •		• •	
if any 1 is G direct sum: false, for all vely there exists unique v $c F(2)$ st. $v = \sum_{i=1}^{m} \frac{1}{2}$	 V:	• •	. ø	• •	• •	• •	
all are false $(d) = -\sum_{i=1}^{m} x_i(\lambda_i)$			•	• •	• •	• •	
$(0) (1 \leq 1) (1 \leq 1) $					• •		
(e) χ_T has n zeroes (With counted multiplicity) and	γ τ ();	i) = α _i	(λ _i)	pr eac	hj.	• •	
We will prove:	• •	• •	. o	• •	• •	• •	• • •
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Kue's Proof.							
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	• •		• •	• •	• •	• •	· · ·
$[If V]$ has a basis of eigenvectors of T, then $V = \bigoplus_{i=1}^{\infty} E_T(X_i)$							
Proof: Assume V has a basis of eigenvectors of T, Call	ił B=	{ v .,ı	, V _{1,2} , \Γ		κ, V ₂ ,ι,	, V ₂ ,	K,, / .
grouped by eigenvalues: Vi,j is the j-th eigenvector for eigenvalue λ;	K _i =	# of	basis	vectors	belonging	to eig	lenvalue λ;
Need to show: \bigcirc For all $v \in V$, $\exists v_i \in E_T(\lambda_i)$ s.t. $v = v_1 + \dots + V_m$	• •		•	• •	· ·	• •	· · ·
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Friday									
Brandon's (b) ⇒ V hqs	(a) a basis	of eigenve	ctors of T	⇒ T is diag	ronalizable.	· · · · · ·	· · · ·	· · ·	· · · · · ·
V has	a basis ; _B [T] _B	[K,, Vm] = [_B TV ₁	} В is вТVm	$A basis;$ $a = (\beta \lambda_i V_i)$	τν; = μ;ν; Β ληνη	$= \begin{bmatrix} \mu_i & 0 \\ 0 \\ \vdots & \ddots \\ 0 \end{bmatrix}$	0 0 1 0 1 0	T is iagonal	· · · · · ·
Kyle (c) = Ε _τ () Β: =	+ Eric; > (d) };) {V:1	Vim ²	dim(E _T)=m	dim (E)=Y-	· · · ·	Proof in pro	· · · · · · · · · · · · · · · · · · ·	 	
Kye's	Suppor	Tf 1 = 1	· · · · ·			Idea: try to Bu={VV	find a bas	is of U.	· · · · · ·
		dim	11 = dim V	+ dim W		$B_{W} = \{w_{1},, W = A_{V}, V +, W = B_{V}, W +, W = B_{V}, W +, W = B_{V}, W = M_{V}, W = M$	ωm} + αnVn + βn Ub	· · · ·	· · · · · ·
· · ·	· · ·	∀ue(so tha U: where,),	ωε W:	· · ·	· · · · · ·	· · · ·	· · ·	
· · ·	· · ·	v, n V+u	υ are Uniq υ = α, V1 +	9ue +α _n Vn+β,ω, +	+ Bm Wm =	· · · · · · · · · · · · · · · · · · ·	· · · ·	· · ·	· · · · · ·
Kyle	if U= then V/	V⊕W NW=∮	contains 0	· · · · ·	· · ·	 	· · · ·	 	
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