WEEK 1 REVIEW

Def A GROUP is a set G w/ an operation $*: G \times G \to G$ s.t.

$\forall a,b,c$ $\quad (a*b)*c = a*(b*c)$ [assoc.]

$\exists e\ \forall a$ $\quad e*a = a*e = a$ [id.]

$\forall a\ \exists b$ $\quad a*b = b*a = e$ [inv.]

If we also have

$\forall a,b$ $\quad a*b = b*a$ [comm.]

we call it a COMMUTATIVE GROUP.

Ex

$(\mathbb{R}, +)$ $\quad e=0$

$(\mathbb{R}\setminus\{0\}, \cdot)$ $\quad e=1$

$(\mathbb{R}^2, +)$ $\quad e=(0,0)$

$(\mathbb{Z}, +)$ $\quad e=0$

Def A FIELD is a set F w/ two ops. $+ : F \times F \to F$

$\cdot : F \times F \to F$

s.t. $(F, +)$ is a comm. gp. w/ id. e_+

$(F \setminus \{e_+\}, \cdot)$ is a comm. gp. w/ id. $e_\cdot$

and

$\forall a,b,c \quad a \cdot (b+c) = (a \cdot b) + (a \cdot c)$

Rmk We usually write e_+ as 0 and $e_\cdot$ as 1 (by analogy w/ numbers).

Ex $(\mathbb{R}, +, \cdot)$

$(\mathbb{C}, +, \cdot)$

$(\mathbb{Q}, +, \cdot)$

$(\mathbb{F}_2, +_2, \cdot_2)$

Def A VECTOR SPACE over a field F is a set V w/ an op. $\oplus : V \times V \to V$ and a func. $\odot : F \times V \to V$ s.t. $(V, \oplus)$ is a comm. gp. w/ id. $e_\oplus$

$\forall \alpha, \beta \ \forall v \quad \alpha \odot (\beta \odot v) = (\alpha \cdot \beta) \odot v$

$\forall v \quad 1 \odot v = v$

$\forall \alpha, \beta \ \forall v \quad (\alpha + \beta) \odot v = (\alpha \odot v) \oplus (\beta \odot v)$

$\forall \alpha \ \forall v, w \quad \alpha \odot (v \oplus w) = (\alpha \odot v) \oplus (\alpha \odot w)$

Elt's of V are called VECTORS

" " F " " SCALARS.

Rmk We usually write $e_\oplus, \oplus, \odot$ as $0, +, \cdot$ (just be careful not to confuse scalars and vectors!)

Def A SUBSPACE U of a vec. sp. V is a subset that is also a vec. sp. w/ the same operations.

Prop If $U \subseteq V$ is such that

$0 \in U$

$\forall u_1, u_2 \quad u_1 + u_2 \in U$

$\forall \alpha \, \forall u \quad \alpha u \in U$

then U is a subsp. of V.

In other words, given a subset of V, we only need to check that it contains 0 and is "closed" under $+$ and $\cdot$!