# MATH 115A
## Linear Algebra
Spring 2025

## General information

| | | Lecture | Discussion |
|---|---|---|---|
| **Section** | Time | MWF 12:00–12:50 | TR 12:00–12:50 |
| | Location | BOELTER 5440 (MW), MS 6627 (F) | BOELTER 5440 |
| **Instructor** | Name | Nicholas Hu (he/him) | Kye Shi (he/they) |
| | Email | `njhu@math.ucla.edu` | `kwshi@math.ucla.edu` |
| | Website | `njhu.ca` | `kye.sh` |
| **Office hours** | Time | MWF 13:30–14:30 or by appt. | R 13:00–15:00 |
| | Location | MS 6147 | KAUFMAN 136 |

### Acknowledgment

UCLA occupies land on the traditional, ancestral, and unceded territory of the Tongva/Kizh (Gabrieleño) people, who are the past, present, and future caretakers of Tovaangar (the Los Angeles basin and South Channel Islands).[1]

Settler colonialism continues to structure the relationship of this university and its community to the Indigenous peoples of this continent. As you take this course, we encourage you to consider the ways in which mathematics in particular has been and is still used in processes of settler colonialism and imperialism throughout the world – from the cultural supersession of Indigenous conceptions of quantity and space, to the material production of technologies for dispossession, exploitation, and elimination.[2] We also urge you to reflect on how this might inform your practice of mathematics or science beyond this course and in your future profession.

## Course content

This course is an introduction to *linear algebra*, the study of vector spaces and linear maps between them. The origins of linear algebra can be traced back to the ancient Babylonian and Chinese civilizations that developed methods for solving systems of linear equations. Its concepts have since been greatly abstracted and generalized, and it is now central to many areas of modern mathematics and ubiquitous in science, technology, and engineering.

A detailed outline of topics is given in the Schedule section below.

[1] For more information about local Indigenous nations, see `gabrielinotongva.org`, `gabrieleno-nsn.us`, and `gabrielenoindians.org`. For more information about Indigenous nations globally, see `native-land.ca`.

[2] Some materials that might provide starting points for further consideration are:
- S. Englert. *Settler Colonialism: An Introduction*. Pluto Press, 2022.
- A. J. Bishop. "Western mathematics: the secret weapon of cultural imperialism". *Race & Class* 32.2 (1990), pp. 51–65.
- `magazine.scienceforthepeople.org`
- `www.justmathematicscollective.net/resources.html`

### Description

Nearly two thousand years ago, Chinese mathematicians considered the following linear algebra problem [Har11]:

> Given 3 bundles of superior paddy [unhusked rice], 2 bundles of ordinary paddy, and 1 bundle of inferior paddy, [together they yield] 39 *dou* [unit of volume] of grain; 2 bundles of superior paddy, 3 bundles of ordinary paddy, and 1 bundle of inferior paddy [together yield] 34 *dou* of grain; 1 bundle of superior paddy, 2 bundles of ordinary paddy, and 3 bundles of inferior paddy [together yield] 26 *dou* of grain.
>
> Problem: 1 bundle of superior, ordinary, and inferior paddy each yield how much grain?

Nowadays, linear algebra has an enormous number of applications, such as cryptography, machine learning, computer graphics, search engines, signal processing, quantum mechanics, electrical networks, natural language processing, image compression, portfolio optimization, medical imaging, recommendation systems, and so much more! How did the concepts and methods studied by these humble farmers become applicable to fields other than those of rice?

The key lay in realizing the underlying patterns and structures that these different settings and problems had in common. In this course, we will develop the theory of linear algebra from the ground up, starting from foundational definitions and concepts and working our way up to more complex theorems, ensuring that we understand and rigorously justify each step. With this groundwork, you will be well-equipped to apply this theory to a wide variety of disciplines. In addition, you will hopefully acquire the capability to formulate and analyze problems accurately, precisely, and thoroughly – an ability that is not only fundamental to the discipline of mathematics, but also a powerful way of reasoning about the world.

### Requisites

- **MATH 33A** – Linear Algebra and Applications (prerequisite)

  In MATH 115A, we will cover many topics that will likely be familiar to you if you have taken MATH 33A, such as subspaces, linear transformations, eigenvalues, and orthogonality. However, we will study them with much greater abstraction and generality and will develop the theory of linear algebra from first principles. Thus, while MATH 33A would undoubtedly be useful for providing motivation, context, intuition, and applications for the concepts in this course, it is not conceptually or logically necessary for MATH 115A, and knowledge from MATH 33A will not always be directly applicable to MATH 115A.

- Mathematical proof (unofficially recommended)

  The most important requisite for this course, while not officially or explicitly stated elsewhere, is a working knowledge of mathematical proof – namely, familiarity with sets, logic, and proof techniques (e.g., direct proof, proof by contradiction, proof by contraposition, proof by induction). While these concepts will be briefly reviewed and discussed throughout this course, the current department curriculum unfortunately does not allow for a detailed treatment of these topics. Consequently, you should be prepared to review or learn them implicitly through the proofs in this course or explicitly through self-study outside of class time (for recommended resources, see the Materials section).

### Materials

The official course textbook is Friedberg, Insel, and Spence's *Linear Algebra* [FIS18]. However, I will give my own exposition and interpretation of the material, drawing on other texts such as that of Axler [Axl24]. The content of introductory linear algebra courses is fairly standard and there is a plethora of texts on this subject (feel free to ask for recommendations). To review or learn about mathematical proof specifically, Hammack's [Ham18] book is one possible source.

## Tips for success

- Read this syllabus!

- Attend class (both lectures and discussions).
- Pay attention to your instructors and classmates in class.
- Participate in the learning activities that occur in class.
- Ask questions in class, in office hours, and on Campuswire.
- Take notes in class and review them afterwards.
- Read the recommended references.
- Complete the learning assessments, especially the homework assignments.
- Collaborate and study with your classmates.
- Consider and seek feedback from your instructors and classmates, especially about errors you might have made.
- Ask for help from your instructors and classmates.
- Take care of your physical, mental, and emotional well-being!

## Asking questions

Office hours are the best times to ask any course-related questions, and we also have a Campuswire classroom! Here are some examples of things you could ask about in office hours or elsewhere, and how you should ask.

- Homework

  If you need help with a homework problem, you should make a reasonable attempt to solve the problem beforehand and should ask *specific* questions about it. For example, rather than ask "Can you help me with this problem?", you should ask "I am trying to do $A$ and have done $B$. I believe the next step is $C$, but I am having difficulty with this step. Can you help me with it?"

- Lectures

  As for homework, your questions should be specific. For example, rather than ask "I don't understand this proof. Can you explain it?", you should ask "I don't understand how we went from step $A$ to step $B$. Can you explain this step?"

- Feedback

  You are welcome to ask for feedback on your past and present assessments. However, note that the instructors will not assess your work comprehensively or definitively before it is submitted, as this would defeat the purpose of the assessment. For instance, we will not answer questions such as "Can you check that my proof is correct?"; however, we may answer questions such as "I believe I made an error in going from step $A$ to step $B$ because of $C$, but I am not sure what it is. Can you help me find it?"

- References

  Feel free to ask about the recommended references or for additional references.

- Course policies

  Feel free to ask for clarification about any course policies.

- Accommodations and difficulties

  If you need or desire accommodations, or are experiencing difficulties in this course, please refer to the Contact and Accommodations sections.

## Learning objectives

By the end of this course, you will be able to:

- *Define* objects in linear algebra (e.g., linear maps, eigenvalues).
- *State* theorems in linear algebra (e.g., the rank-nullity theorem, the spectral theorem).
- *Give examples and non-examples of* objects in linear algebra by verifying their definitions.
- *Apply* theorems in linear algebra by verifying their hypotheses and expressing their conclusions.
- *Explain* proofs of theorems in linear algebra by describing the strategies and techniques used and how relevant definitions and hypotheses are used.
- *Prove* results in linear algebra by synthesizing known facts, results, and techniques.

In addition, you will be able to:

- *Evaluate* the correctness and quality of your work and your peers' work by engaging in reflection and group activities throughout the course.
- *Recognize the value of* writing and communication in mathematics by engaging in reflection and group activities throughout the course.

More detailed weekly learning objectives will be posted on Bruin Learn.

## Assessments

The assessments in this course are outlined below. Further details about these assessments, such as instructions and grading rubrics, will be provided as they arise.

### Participation

- This course is designed to reflect the collaborative nature of mathematics. To this end, you will be asked to *actively* participate in learning activities during class (both lectures and discussions).
- Participation includes (but is not limited to): asking *and* answering questions, sharing your thoughts during brainstorming and problem-solving sessions, and presenting results and write-ups to your classmates.

### Reflections and retrospective

- There will be brief written reflections at the beginning and at the end of the course about your mathematical background, goals, views, etc.
- You will also schedule a retrospective meeting with us during Week 11 of the quarter (finals week), in which we will discuss your reflections and your performance in the course.

### Scribing

- Each week, a group of up to 4 students (the "scribes") will be tasked with taking notes on everything that happens in class (both lecture and discussions) that week. You will sign up to scribe *once* during the course.
- Scribes should coordinate and collaborate with each other throughout the week and must submit a *single, jointly created* set of notes, due in the same week on **Friday** at **23:59** on Gradescope.
- The TA may request revisions, due the **following Friday** (i.e., 7 days later) at **23:59** on Gradescope. The final version of the notes will then be posted online.

### Homework assignments

- Homework will be assigned each Friday and will be due the following **Friday** at **23:59**, on Gradescope.
- Collaboration with your classmates on problem-solving is encouraged, but you must write up your submission individually.

### Quizzes

- There will be a quiz each **Friday** during part of the lecture (for approximately 20 minutes).
- No resources will be permitted.

### Final exam

- The final exam will be on **Wednesday, June 11$^{\text{th}}$** from **15:00** to **18:00**, in BOELTER 2760.
- The only resource permitted will be a ***single-sided* 8.5 in by 11 in (215.9 mm by 279.4 mm)** sheet of notes (written or typed).

### Surveys

- There will be optional online surveys for an education research study at the beginning, middle, and end of the course, as well as LA feedback surveys at the middle and end of the course.
- You will be awarded bonus credit for each completed survey as specified below.

## Grading

The contributions of the assessments to your final grade are as follows.

| | Quantity | Weight |
|---|---|---|
| Participation | – | 10% |
| Reflections and retrospective | 2 | 6% |
| Scribing | 1 | 6% |
| Homework assignments | *top 8 of* 9 | 24% |
| Quizzes | *top 8 of* 10 | 24% |
| Final exam | 1 | 30% |
| Surveys (*bonus*) | $n$ | $0.5n\%$ |

Your final percentage grade will be converted to a final letter grade according to the table below.

| | **F** | **D−** | **D** | **D+** | **C−** | **C** | **C+** | **B−** | **B** | **B+** | **A−** | **A** | **A+** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Minimum percentage** | 0 | 55 | 58 | 62 | 65 | 68 | 72 | 75 | 78 | 82 | 85 | 88 | 92 |

I reserve the right to scale grades as warranted. You may also be asked to meet with me to explain any of your submitted work. Such a meeting will be mandatory and part of the assessment of the quality of the work.

## Schedule

The following schedule is tentative and subject to change.

| Week | Topics and references | Assessments |
|---|---|---|
| **1**<br>Mar 31 –<br>Apr 04 | Vector spaces<br>• Fields<br>• Vector spaces<br>• Subspaces<br>[FIS18, ss. 1.1, 1.2, 1.3; Axl24, ss. 1A, 1B, 1C] | F: Quiz 1 |
| **2**<br>Apr 07 –<br>Apr 11 | Vector spaces<br>• Linear independence<br>• Span<br>• Bases and dimension<br>[FIS18, ss. 1.4, 1.5, 1.6; Axl24, ss. 2A, 2B, 2C] | F: Quiz 2<br>F: HW 1 due |
| **3**<br>Apr 14 –<br>Apr 18 | Linear maps<br>• Linear maps<br>• Kernel and image<br>[FIS18, ss. 2.1, 2.3; Axl24, ss. 3A, 3B] | F: Quiz 3<br>F: HW 2 due |
| **4**<br>Apr 21 –<br>Apr 25 | Linear maps<br>• Isomorphisms and change of bases<br>• Matrices of linear maps<br>• Matrix operations<br>[FIS18, ss. 2.2, 2.4, 2.5; Axl24, ss. 3C, 3D] | F: Quiz 4<br>F: HW 3 due |
| **5**<br>Apr 28 –<br>May 02 | (Interlude)<br>• Determinants<br>• Polynomials<br>[FIS18, ch. 4; Axl24, ch. 4, s. 9C] | F: Quiz 5<br>F: HW 4 due |
| **6**<br>May 05 –<br>May 09 | Eigenvalues and eigenvectors<br>• Eigenvalues and eigenvectors<br>• Minimal and characteristic polynomials<br>[FIS18, ss. 5.1; Axl24, ss. 5A, 5B] | F: Quiz 6<br>F: HW 5 due |
| **7**<br>May 12 –<br>May 16 | Eigenvalues and eigenvectors<br>• Triangularization<br>• Diagonalization<br>[FIS18, ss. 5.2; Axl24, ss. 5C, 5D] | F: Quiz 7<br>F: HW 6 due |
| **8**<br>May 19 –<br>May 23 | Inner product spaces<br>• Inner product spaces<br>• Orthogonality and orthonormal bases<br>• Orthogonal projections and least squares<br>[FIS18, ss. 6.1, 6.2, 6.6; Axl24, ss. 6A, 6B, 6C] | F: Quiz 8<br>F: HW 7 due |
| **9**<br>May 26 –<br>May 30 | Adjoints<br>• Adjoints<br>• Normal, self-adjoint, unitary, and positive operators<br>[FIS18, ss. 6.3, 6.4, 6.5; Axl24, ss. 7A, 7C, 7D] | F: Quiz 9<br>F: HW 8 due |
| **10**<br>Jun 02 –<br>Jun 06 | The spectral theorem and singular value decomposition<br>• The spectral theorem<br>• The singular value decomposition<br>[FIS18, ss. 6.6, 6.7; Axl24, ss. 7B, 7E] | F: Quiz 10<br>F: HW 9 due |

# Policies

## Contact

- For questions of general interest, especially mathematical ones, please post in our Campuswire classroom using the "Post to everyone" setting.
- For moderately personal or confidential questions, please post in our Campuswire classroom using the "Post to instructors & TAs" setting.
- For highly personal or confidential questions, please send me an email and include your name and UID.

## Attendance and participation

While attendance in lecture and discussion is not mandatory and will not be *directly* assessed, participation will be a component of your grade and certain assessments (e.g., quizzes) will take place in class. In addition, we believe that the activities offered in class will enhance your learning experience and, as such, strongly encourage you to attend and participate in them.

## Late or missed assessments

Credit will not be awarded for late or missed assessments except as specified in the Grading and Accommodations sections.

## Accommodations

Students needing accommodations based on a disability should first contact the Center for Accessible Education (CAE) (A255 Murphy Hall, `caeintake@saonet.ucla.edu`, 310-825-1501). All students are welcome to contact me to request unofficial or informal accommodations for reasons including (but not limited to) athletic events, emergencies, health and safety concerns, and religious observances. Any requests for accommodations should be made as early as possible to maximize the number of options available and their effectiveness, especially for those from the CAE (which should be made within the first two weeks of the quarter as per their recommendation).

## Academic integrity

The regulations in Section 102.01 of the UCLA Student Conduct Code apply to this course. In particular, all work (including words and ideas) that is not your own must be appropriately attributed or cited; this includes that of non-human sources such as generative artificial intelligence. Furthermore, regardless of the sources you use, the exposition of your work must be original; it must consist of your own wording and phrasing and cannot be a copy or trivial modification of material from other sources (e.g., merely changing variable names or permuting sentences).

No specific citation style or format will be required, but enough information must be provided for others to easily locate and access the sources you used. For instance, an author, book (and edition if applicable), and page number; or a website URL; or the name and contact information of a classmate.

If you find these regulations unclear or difficult to abide by for any reason, please contact me so that I can clarify them or attempt to assist you in your situation.

## Title IX

Title IX prohibits gender discrimination, including sexual harassment, domestic and dating violence, sexual assault, and stalking. If you have experienced sexual harassment or sexual violence, you can receive confidential support and advocacy at the CARE Advocacy Office for Sexual and Gender-Based Violence (330 De Neve Dr., `CAREadvocate@careprogram.ucla.edu`, 310-206-2465). In addition, Counseling and Psychological Services (CAPS) provides confidential counseling to all students and can be reached 24/7 at 310-825-0768. You can also report

sexual violence or sexual harassment directly to the University's Title IX Coordinator (2241 Murphy Hall, `titleix@equity.ucla.edu`, 310-206-3417).

## Resources

If you experience difficulties of any kind while taking this course, whether physical, mental, emotional, financial, or academic, you are welcome to contact me and I will try my best to help you. Please note that I am required under the UC Policy on Sexual Violence and Sexual Harassment to inform the Title IX Coordinator should I become aware that you or any other student has experienced sexual violence, sexual harassment, or other prohibited conduct.

An extensive directory of resources available to students can be found at `bewellbruin.ucla.edu`.

## References

[Axl24] S. Axler. *Linear algebra done right*. 4th ed. Springer, 2024.
[Eng22] S. Englert. *Settler Colonialism: An Introduction*. Pluto Press, 2022.
[FIS18] S. H. Friedberg, A. J. Insel, and L. E. Spence. *Linear Algebra*. 5th ed. Pearson, 2018.
[Ham18] R. Hammack. *Book of Proof*. 2018.
[Har11] R. Hart. *The Chinese Roots of Linear Algebra*. Johns Hopkins University Press, 2011.