Worksheet 11

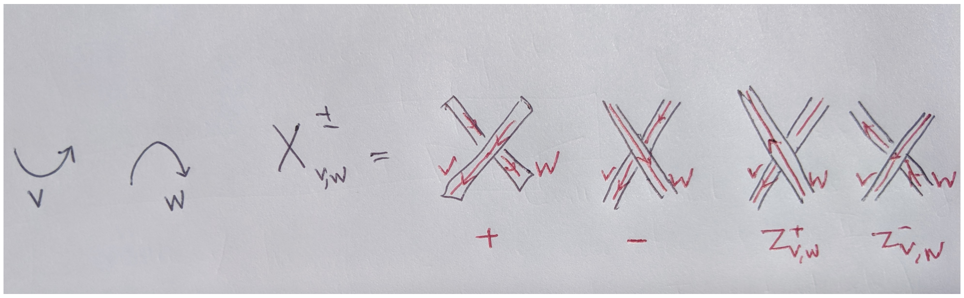
July 30, 2019

1. Recall that an HCDR graph is **homogeneous** if it intersects the planes $(z = 0)$ and $(z = 1)$ with the positively-oriented side face-up, following Reshetikin–Turaev's definition. Show that every homogeneous HCDR graph can be made to have all twists occuring in type 1 bands ("coupons").

   Remark 1. *I'm still not clear if this fixes the potential issues with Reshetikin–Turaev's definition of HCDR(A). Essentially, with this fix we have reduced issues of generating twists to generating those which occur on coupons; however, all generators for the morphisms in HCDR(A) which include coupons are "flat" in an isotopy-invariant sense (see pictures of $\Gamma(f, \eta, \eta')$ for $\eta, \eta' \in N(A)$ and $f\colon \eta \to \eta'$.) So it still seems that you need to include more generators to get any twists in HCDR(A). However, I am quite confident that the arguments given in class work under the restriction to twist-free HCDR-graphs. I encourage you to think more about this issue/ let me know if you come up with a resolution or conceptual explanation. For example, an assertion might be that F such as in the main theorem 5.1 is uniquely determined under the additional hypothesis that it is invariant under "twists of coupons".*

   Remark 2. *I now think this is probably false. But still think about when you can or cannot "transfer twists to coupons".*

2. Let A be a Hopf algebra. An A-colored *HCDR*-graph is said to be even if it is in the collection of A-colored *HCDR* graphs generated (under the monoidal product and composition in *HCDR*(A)) by the collection of graphs $\Gamma(f, \eta, \eta')$ (recall these are the generators needed to get type 1 graphs), combined with the type 2 only *HCDR* graphs:



Show that the value of F determined by a ribbon Hopf algebra structure (A, R, ν) on A is independent of ν. That is, if F' is determined by (A, R, ν') another RHA structure on on A, the $F(g) = F'(g)$ for g an even *HCDR* graph.

(Hint: you have a set of generators for the class of morphisms that you wish to show F and F' agree upon!)

3. Compute $\mathfrak{sl}_2(\mathbb{R})$. (Or conjecture what it should be if you're feeling lazy.)

4. In class, we discussed that the quantized universal enveloping algebra of a Lie algebra has a ribbon hopf algebra structure. Why does this allow you to conclude that the (ordinary) universal enveloping algebra does, too?

5. In Kassel's book, chapter VI, he discussing the quantum enveloping algebra of $\mathfrak{sl}_2$, the main case of interest to us. In fact, much of his book is devoted to developing knot invariants in a way akin to that discussed in my part of this class (it might be a good next step if you find this stuff cool!). Starting on page 121, read Kassel's book and understand his discussion of $U_q(\mathfrak{sl}_2)$. Show that his definition corresponds to the Reshetikin-Turaev definition under the change of variables $q = e^h$: we get an algebra homomorphism $U_q(\mathfrak{sl}_2) \to U_h(\mathfrak{sl}_2)$ which maps the generators of the domain to generators of the codomain, when it is viewed as a topological algebra over $\mathbb{C}[[h]]$ complete with respect to h.