Research Highlights

Holomorphic Maps

These are a collection of questions about non-constant maps from **C** or **C**^n to various geometric spaces. The original result is Picard's Theorem that there are no non-constant maps from **C** to **CP**^1- 3 points. I solved a conjecture of S.S. Chern on a natural generalization of Picard's theorem about holomorphic maps to the complement of n+2 hyperplanes in general position in **CP**^n, which was one of ten problems in geometry that he posed at the International Congress of Mathematicians in Nice in 1970 (solved independently by Hirotaka Fujimoto.)

Hyperbolicity is a quantitative measure, invented by Kobayashi, of how close one can come to having a non-constant map from **C** to a given space. Using a method of R. Brody, I proved that the complement of 2n+1 hyperplanes in general position in **CP**^n is hyperbolic and hyperbolically embedded, and also that a subvariety of a complex torus is hyperbolic if and only if it does not contain a complex subtorus, which was part of a conjecture of A. Bloch from the 1920's.

Phillip Griffiths and I proved Bloch's Conjecture by introducing the notion of jet differentials, we also proved jet differentials they exist in abundance for surfaces of general type. Fairly recently, this result was extended by Demailly using new analytic techniques to all dimensions. Griffiths and I proposed what is now known as the Green-Griffiths Conjecture about holomorphic maps to complements of divisors in algebraic varieties; this remains unsolved.

Differential Invariants

The classical geometry of curves in **R**^3 describes them using three invariants: velocity, curvature and torsion, which involve derivatives of order 1, 2 and 3 respectively. These are examples of differential invariants. For curves in homogeneous spaces G/H, I found a formula that calculates the number of differential invariants of curves on homogeneous spaces involving a given number derivatives for general G and H, relating to E. Cartan's method of the moving frame.

Algebraic Cycles and Hodge Theory

In a paper with Jim Carlson, Joe Harris and Phillip Griffiths, we introduced the notion of infinitesimal variation of Hodge structure and found some nice geometric consequences—for example, an infinitesimal proof of the Noether-Lefschetz Theorem. This paper is the first place where the idea of iteration of the period map is introduced—in physics, this is known as the Yukawa coupling.

I solved a conjecture of Griffiths and Harris on the minimal codimension of the Noether-Lefschetz locus, and suggested an argument for proving that the Noether-lefschetz locus is dense in the classical topology.

I solved a conjecture going back to Riemann known as "quadrics of rank four," showing that the quadratic ideal of a canonical curve is spanned by tangent cones at double points of the Theta-divisor of the Jacobian of the curve.

I created a suite of methods using Koszul cohomology for studying the geometry of varieties. This gave a number of nice geometric results, and led to a conjecture relating the minimal free resolution of canonical curves to the most exceptional linear series that they carry (their Clifford index)—this has become known as Green's Conjecture. With Rob Lazarsfeld, I proved one direction. A joint paper with Lazarsfeld established a consequence of the conjecture about K3 surfaces. Frank Schreyer proved the other direction for first syzygies. In a tour de force, Claire Voisin proved that the other direction holds for general curves. The full conjecture remains open.

I solved, using a vanishing theorem for Koszul cohomology, a conjecture of Griffiths and Harris on the image of the Abel-Jacobi map for general 3-folds in **CP**^3 of degree at least 6 (solved independently by Claire Voisin).

Phillip Griffiths and I solved an interesting conjecture of Rahul Pandharipande on 0-cycles on products of curves.

With Phillip Griffiths, I formulated a conjecture on singularities of normal functions that is equivalent to the Hodge Conjecture. I still consider this a "live" approach to the Hodge Conjecture.

More recently, Phillip Griffiths and I have written a series of papers on Hodge theory and algebraic cycles, which build a suite of tools for studying limit mixed Hodge structures. Our current work is in collaboration with Radu Laza and Colleen Robles.

<u>Cohomological Support Loci</u>

Ropb Lazarsfeld and I solved a conjecture of Beauville and Catanese on cohomological support loci (which topologically trivial line bundles on varieties have non-vanishing cohomology), and found what was then a surprising result that these cohomological support loci are translates of complex subtori. This result has led to beautiful generalizations by others, and has been used in the classification of higher-dimensional varieties.

<u>Commutative Algebra</u>

I originally started working in commutative algebra because certain problems came up in Hodge theory, e.g. in the explicit version of the Noether-Lefschetz theorem, that I needed for geometric results. But then I got interested in the subject in its own right.

David Eisenbud and I solved a conjecture of Craig Huneke on the ideals generated by minors of maps in minimal free resolutions.

I solved a conjecture of David Eisenbud, Jee Koh and Mike Stillman relating syzygies of modules and relations of low rank.

In a paper with David Eisenbud and Joe Harris, we formulated some conjectures that would generalize the Cayley-Bacharach Theorem and proved some suggestive cases of the conjecture.

In my work on generic initial ideals, I proved the Crystallization Principle, which is sometimes used by combinatoricists.

<u>Geometry of convex plane curves</u>

With Stanley Osher, I discovered a series of inequalities on integrals of powers of a convex plane curve that generalize a result of Gage on the integral of k^2. Neither of us ever expected to collaborate.

I am a co-author of six books.
--A book on Hodge Theory and Algebraic Cycles, consisting of CIME lectures by myself, Jacob Murre and Claire Voisin.
--A book "Six Lectures on Commutative Algebra" with lectures by Luchezar Avramov, Craig Huneke, Peter Schenzel, Giuseppe Valla and Wolmer Vasconcelos
--A book with Phillip Griffiths and Matt Kerr, developing the general theory of Mumford-Tate groups and domains over the reals, with numerous results and examples.
--A book with Phillip Griffiths on formal deformations of algebraic cycles.
--A book with Phillip Griffiths and Matt Kerr on special values of automorphic cohomology classes
--A book with Phillip Griffiths and Matt Kerr on Hodge Theory, Complex Geometry and Representation Theory.