Math 32B: Vector Fields

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1 Vector fields, divergence, and curl

A *vector field* assigns to each point (x, y, z) a vector

$$\mathbf{F}(x, y, z) \in \mathbb{R}^3.$$

The *divergence* of a vector field $\mathbf{F}$ assigns to each point (x, y, z) a number

$$\text{div } \mathbf{F}(x, y, z) \in \mathbb{R}^3.$$

The *curl* of a vector field $\mathbf{F}$ assigns to each point (x, y, z) a vector

$$\text{curl } \mathbf{F}(x, y, z) \in \mathbb{R}^3.$$

It is useful to think about a vector field as telling you the velocity of a fluid at each point of space. With this perspective, the divergence tells you how much fluid is being "created" at each point, and at each point, the curl tells you the direction around which the fluid is "spinning" the most, and how much it is spinning around this direction (the magnitude of the curl).

"Creation" is regarded as positive so that when more flows out of a point than into a point the divergence is positive.

This assignment concerns some vector fields which do not depend on z, and have no z-component, i.e. those of the form

$$\mathbf{F}(x, y, z) = (F_1(x, y), F_2(x, y), 0).$$

In this case, curl $\mathbf{F}(x, y, z) = \xi(x, y)(0, 0, 1)$, i.e. the fluid spins around the direction coming out of the page. We use the right-hand rule, so that when the spinning is counterclockwise $\xi(x, y) > 0$; when the spinning is clockwise $\xi(x, y) < 0$.

2 The assignment

- Click here: `http://www.falstad.com/vector/index.html`.
- Select "Flat View".
- Select "Floor: no lines".

  You might find "Floor: streamlines" helpful sometimes.
- You will find it useful to alternate between "Display: Particles (Vel.)" and "Display: Curl Detectors."

  Do NOT use "Display: Particles (Force)"

Fill in the following table by using the applet.

$F_1(x,y)$	$F_2(x,y)$	Setup: name	div $\mathbf{F}$	ξ
1	0	one direction	0	0
$-x$	0	linear to y-axis	< 0	0
$-\frac{1}{x}$	0	inverse to y-axis	> 0	0
y	0	$(y,0)$	0	< 0
y^2	0	$(y^2,0)$	0	the opposite sign to y
x	x^2	(x,x^2)	> 0	the same sign as x
$-x$	$-y$	linear radial	< 0	0
$-\frac{x}{r}$	$-\frac{y}{r}$	const radial	< 0	0
$-\frac{x}{r^2}$	$-\frac{y}{r^2}$	1/r single line	0	0
$-\frac{x}{r^3}$	$-\frac{y}{r^3}$	1/r^2 single	> 0	0
$-y$	x	linear rotational	0	> 0
$-\frac{y}{r}$	$\frac{x}{r}$	constant rotational	0	> 0
$-\frac{y}{r^2}$	$\frac{x}{r^2}$	1/r rotational	0	0
$-\frac{y}{r^3}$	$\frac{x}{r^3}$	1/r^2 rotational	0	< 0
$x-y$	$x+y$	rotation + expansion	> 0	> 0

The ones I have already filled in should help you.

I am using r for the quantity $\sqrt{x^2+y^2}$, NOT $\sqrt{x^2+y^2+z^2}$: our functions do NOT depend on z in this part of the homework.

The fact that I filled in the boxes for $(F_1,F_2) = -\frac{(x,y)}{r^2}$ is crucial!