SO FAR WE HAVE USED TWO TYPES OF BOUNDARY IN OUR CALCULATIONS :

I) (3) THE BOUNDARY OF A 3-D REGION WHICH IS A SURFACE.

i) THE UNIT BALL B.

$$\{(x,y,z) : x^2+y^2+z^2 \leq 1\}$$

ITS BOUNDARY ∂B IS THE SPHERE

$$\{(x,y,z) : x^2+y^2+z^2 = 1\}.$$

ii) THE UNIT BOX $[0,1]\times[0,1]\times[0,1]$.

ITS BOUNDARY $\partial([0,1]\times[0,1]\times[0,1])$

CONSISTS OF 6 SQUARES.

iii) A SOLID CYLINDER C

$$\{(x,y,z) : x^2+y^2 \leq 1, \quad 0 \leq z \leq 1\}.$$

ITS BOUNDARY ∂C CONSISTS OF

- THE LID $\{(x,y,z) : x^2+y^2 \leq 1, \ z=1\}$
- THE BOTTOM $\{(x,y,z) : x^2+y^2 \leq 1, \ z=0\}$
- THE MIDDLE $\{(x,y,z) : x^2+y^2 = 1, \ 0 \leq z \leq 1\}$

iv) $\{(x,y,z) : 1 \leq x^2+y^2+z^2 \leq 2\}$

II) (1) THE ENDPOINTS OF A CURVE.

i) THE UNIT INTERVAL $[0,1] \times \{0\} \times \{0\}$

$$\{(x,0,0) : 0 \leq x \leq 1\}.$$

ITS BOUNDARY CONSISTS OF THE ENDPOINTS

$(0,0,0)$ AND $(0,0,1)$.

ii) THE UPPER HALF OF THE UNIT CIRCLE IN THE xy-PLANE

$$\{(x,y,0) : x^2+y^2=1, \; y \geqslant 0\}.$$

ITS BOUNDARY CONSISTS OF THE ENDPOINTS

$(-1,0,0)$ AND $(1,0,0)$.

IN EACH CASE WE DEALT WITH ORIENTATIONS.

I) A 3-D REGION IS ORIENTED USING x,y,z.

ITS BOUNDARY SURFACE IS GIVEN THE OUTWARD POINTING NORMAL.

II) A CURVE IS ORIENTED BY SPECIFYING A DIRECTION.

ITS BOUNDARY POINTS ARE ORIENTED BY GIVING
A + TO THE END POINT AND
A − TO THE START POINT.

III) (2) THE BOUNDARY OF A SURFACE WHICH CONSISTS OF CURVES.

i) THE UNIT DISK D IN THE xy-PLANE

$$\{(x,y,0) : x^2+y^2 \leq 1\}.$$

ITS BOUNDARY ∂D IS THE UNIT CIRCLE

$$\{(x,y,0) : x^2+y^2 = 1\}.$$

ii) THE UNIT SQUARE IN THE xy-PLANE $[0,1]\times[0,1]\times\{0\}$.

ITS BOUNDARY IS THE UNION OF 4 LINE SEGMENTS

$$\begin{aligned}\partial\left([0,1]\times[0,1]\times\{0\}\right) = \; & [0,1]\times\{0\}\times\{0\} \;\cup \\ & \{1\}\times[0,1]\times\{0\} \;\cup \\ & [0,1]\times\{1\}\times\{0\} \;\cup \\ & \{0\}\times[0,1]\times\{0\}.\end{aligned}$$

iii) A CYLINDER WITHOUT LID OR BOTTOM

$$\{(x,y,z) : x^2+y^2=1, \; 0 \leq z \leq 1\}.$$

ITS BOUNDARY CONSISTS OF TWO CIRCLES

$$\{(x,y,z) : x^2+y^2=1, \; z=1\}$$
$$\{(x,y,z) : x^2+y^2=1, \; z=0\}.$$

HOW TO DEAL WITH ORIENTATIONS?

THE ORIENTATION OF THE SURFACE IS A CHOICE OF NORMAL.
" " " " CURVE IS A CHOICE OF DIRECTION.

HOW SHOULD THESE CHOICES RELATE?

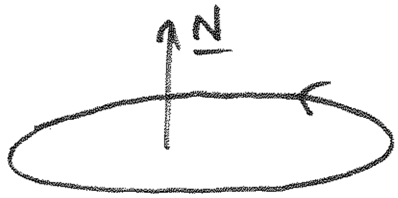
i) PUT YOUR FEET ON THE SURFACE, YOUR HEAD AT THE END OF THE NORMAL VECTOR.

WHEN YOU'RE AT THE BOUNDARY, WALK WITH THE SURFACE ON YOUR <u>LEFT</u>.

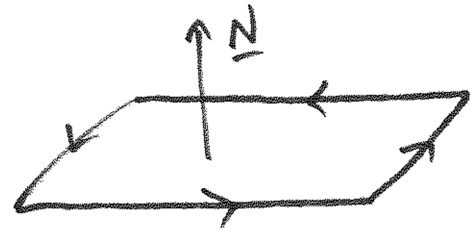
2) STARE DOWN ON THE SURFACE SO THAT THE NORMAL VECTOR POKES YOU IN THE EYE.

DRAW COUNTER-CLOCKWISE CIRCLES.
LET THESE ORIENT THE BOUNDARY.

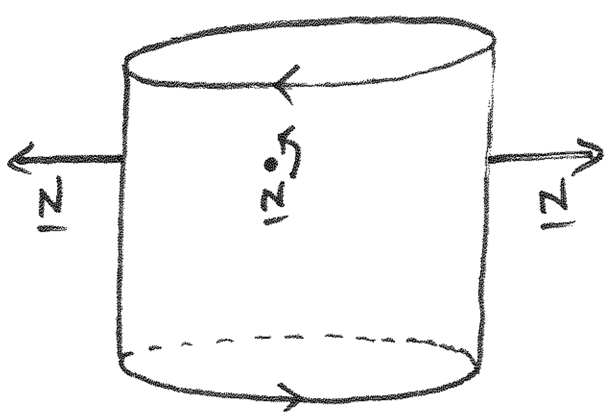
i)



ii)



iii)



STOKES' THM

SUPPOSE S IS AN ORIENTED SURFACE WITH ORIENTED BOUNDARY ∂S.

SUPPOSE $\underline{F}$ IS DIFFERENTIABLE NEAR S. THEN

$$\oint_{\partial S} \underline{F} \cdot d\underline{r} = \iint_{S} \nabla \times \underline{F} \cdot d\underline{S} .$$