LAST TIME :

∂(SURFACE) = CURVES

ORIENTATION OF A SURFACE = CHOICE OF NORMAL
" " " CURVE = " " DIRECTION

HOW TO RELATE THESE CHOICES?

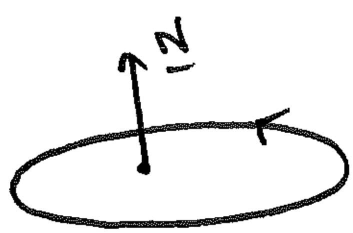
1) FEET ON SURFACE
HEAD AT END OF NORMAL

AT BOUNDARY, WALK W/ SURFACE ON LEFT.

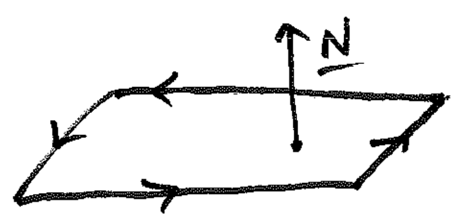
2) STARE DOWN ON SURFACE
NORMAL POKES YOU IN THE EYE

DRAW COUNTER-CLOCKWISE CIRCLES
LET THESE ORIENT BOUNDARY.

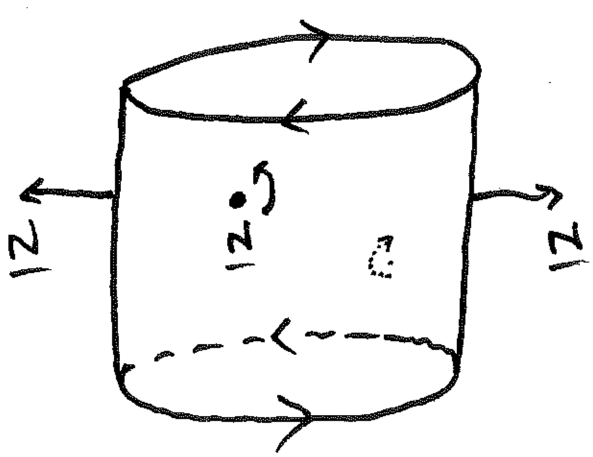
i)



ii)



iii)



STOKES' THM

SUPPOSE S IS AN ORIENTED SURFACE WITH ORIENTED BOUNDARY ∂S.

SUPPOSE $\underline{F}$ IS DIFFERENTIABLE NEAR S.

THEN

$$\oint_{\partial S} \underline{F} \cdot d\underline{r} = \iint_S (\nabla \times \underline{F}) \cdot d\underline{S} .$$

THIS FINALLY TELLS US WHAT CURL IS.

SUPPOSE P IS A POINT,

$D(P)$ IS A FLAT DISK AROUND P, W/ UNIT NORMAL $\underline{n}$.

$$\frac{\oint_{\partial D(P)} \underline{F} \cdot d\underline{r}}{\text{AREA}(D(P))} = \frac{\iint_{D(P)} (\nabla \times \underline{F}) \cdot \underline{n} \, dS}{\iint_{D(P)} 1 \, dS}$$

SO

$$\lim_{\substack{D(P) \text{ SHRINKS} \\ \text{TO } P}} \frac{\oint_{\partial D(P)} \underline{F} \cdot d\underline{r}}{\text{AREA}(D(P))} = (\nabla \times \underline{F})(P) \cdot \underline{n} .$$

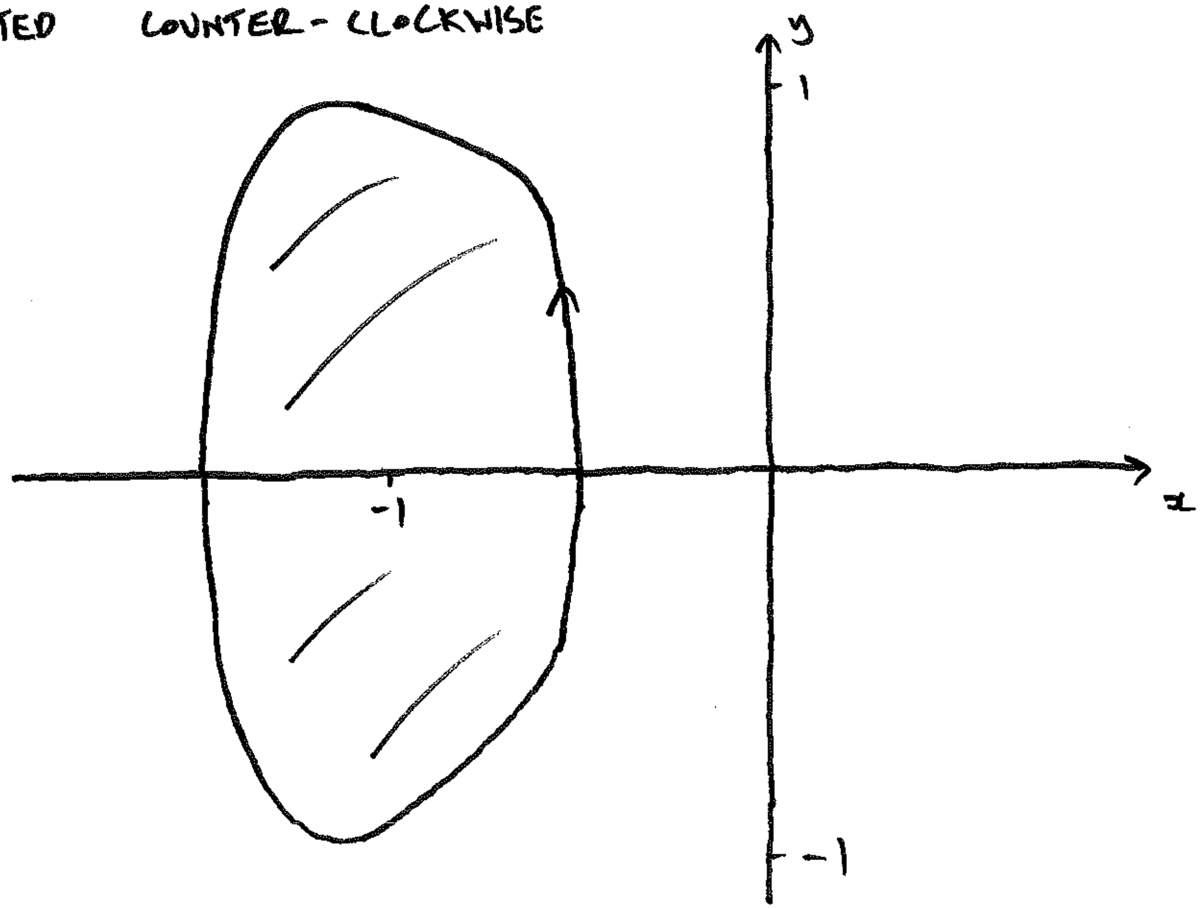
WORK DONE CIRCLING P
SCALED BY AREA.

EXAMPLE

LET C BE THE CURVE IN THE xy-PLANE

$$\left(x+\tfrac{1}{2}\right)^6 + \left(x+\tfrac{1}{2}\right) + y^6 = 0$$

ORIENTED COUNTER-CLOCKWISE



LET $\underline{F} = \dfrac{(-y, x, 0)}{x^2+y^2}$.

CALCULATE $\displaystyle\oint_C \underline{F} \cdot d\underline{r}$.

LET S BE THE SHADED REGION WITH NORMAL POINTING OUT.

THEN $\partial S = C$.

SO

$$\oint_C \underline{F} \cdot d\underline{r} = \oint_{\partial S} \underline{F} \cdot d\underline{r} = \iint_S (\nabla \times \underline{F}) \cdot d\underline{S} = \iint_S \underline{0} \cdot d\underline{S} = 0.$$

RELATED EXAMPLE

LET C BE THE CURVE IN THE xy-PLANE

$$\left(x - \tfrac{1}{2}\right)^6 + \left(x - \tfrac{1}{2}\right) + y^6 = 0$$

ORIENTED COUNTER-CLOCKWISE.

LET $\underline{F} = \dfrac{(-y, x, 0)}{x^2 + y^2}$.

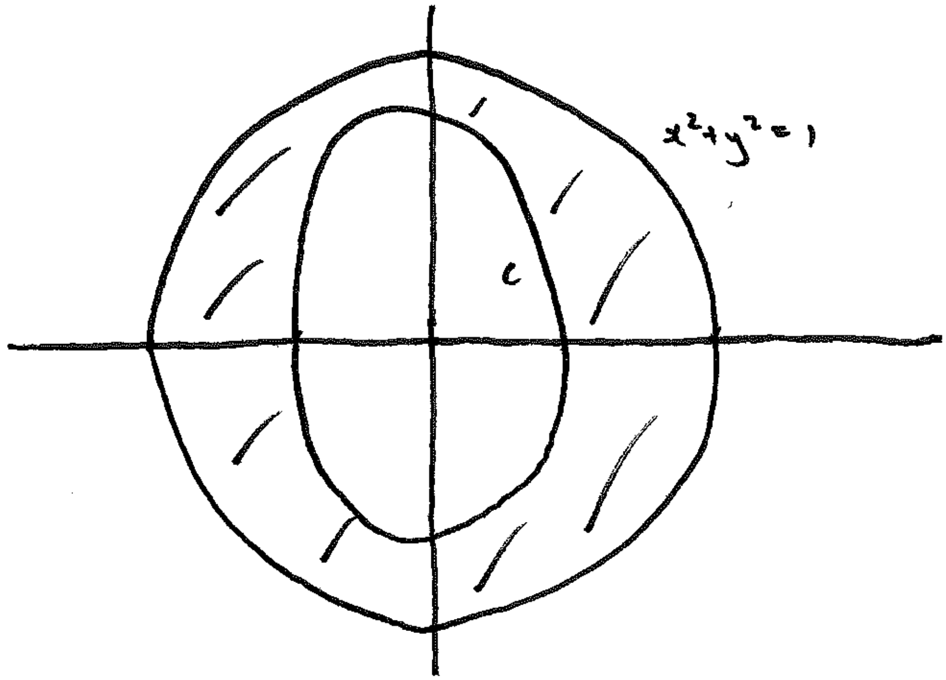
CALCULATE $\oint_C \underline{F} \cdot d\underline{r}$.

LET S BE THE SURFACE BETWEEN C AND THE UNIT CIRCLE

$$x^2 + y^2 = 1, \quad z = 0,$$

WITH OUTWARD POINTING NORMAL.

$$\oint_{\partial S} \underline{F} \cdot d\underline{r} = \iint_S (\nabla \times \underline{F}) \cdot d\underline{S} = \iint_S \underline{0} \cdot d\underline{S} = 0.$$



PLOT USING GRAPHING SOFTWARE FOR BETTER PICTURE!

∂S = UNIT CIRCLE ORIENTED COUNTER-CLOCKWISE $\cup$ C ORIENTED CLOCKWISE, ie NEGATIVE OF QUESTION.

So
$$\oint_C \underline{F} \cdot d\underline{r} = \oint_{\substack{\text{UNIT} \\ \text{CIRCLE} \\ \circlearrowleft}} \underline{F} \cdot d\underline{r} = 2\pi.$$