SCALAR-VALUED FUNCTIONS $\xrightarrow[d]{\text{GRAD}}$ VECTOR FIELDS $\xrightarrow[d]{\text{CURL}}$ VECTOR FIELDS $\xrightarrow[d]{\text{DIV}}$ SCALAR-VALUED FUNCTIONS

$f \longmapsto \nabla f$ $\qquad$ $\underline{F} \longmapsto \nabla\times\underline{F}$ $\qquad$ $\underline{F} \longmapsto \nabla\cdot\underline{F}$

$\underline{F} = \nabla f$ IS PATH-INDEPENDENT, ALL ITS CIRCULATIONS ARE 0.

$\underline{F} = \nabla\times\underline{A}$ IS SURFACE-INDEP., ITS FLUX THROUGH A CLOSED SURFACE IS 0.

FT LINE INTEGRALS

$$\boxed{\text{PLUG IN } \pm f(P).} \qquad f(Q) - f(P) = \int_C \nabla f \cdot d\underline{r} \qquad \boxed{\int_C \underline{F}\cdot d\underline{r}}$$

STOKES'

$$\oint_{\partial S} \underline{F}\cdot d\underline{r} = \iint_S (\nabla\times\underline{F})\cdot d\underline{S} \qquad \boxed{\iint_S \underline{F}\cdot d\underline{S}}$$

GAUSS'

$$\iint_{\partial E} \underline{F}\cdot d\underline{S} = \iiint_E \nabla\cdot\underline{F}\, dV \qquad \boxed{\iiint_E f\, dV}$$

$\nabla f = 0 \Rightarrow f(P) = f(Q)$ WHEN P, Q ARE CONNECTED BY A PATH ON WHICH f IS DEFINED

$\nabla\times\underline{F} = 0 \Rightarrow \oint_{C_1} \underline{F}\cdot d\underline{r} = \oint_{C_2} \underline{F}\cdot d\underline{r}$ WHEN $(-C_1)\cup C_2$ IS A BOUNDARY OF A SURFACE ON WHICH $\underline{F}$ IS DEFINED.

$\nabla\cdot\underline{F} = 0 \Rightarrow \iint_{S_1} \underline{F}\cdot d\underline{S} = \iint_{S_2} \underline{F}\cdot d\underline{S}$ WHEN $(-S_1)\cup S_2$ IS A BOUNDARY OF A REGION ON WHICH $\underline{F}$ IS DEFINED.

$Q - P \longleftarrow C$ $\qquad$ $\partial S \longleftarrow S$ $\qquad$ $\partial E \longleftarrow E$

ORIENTED POINTS (0D) $\xleftarrow{\partial}$ ORIENTED CURVES (1D) $\xleftarrow{\partial}$ ORIENTED SURFACES (2D) $\xleftarrow{\partial}$ 3D REGIONS

ENDPOINT − START POINT $\qquad$ BOUNDARY CURVE W/ INDUCED ORIENTATION $\qquad$ ENCLOSING SURFACE W/ OUTWARD NORMAL