Math 32B: $d\theta$

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1 Another problem

Let $f(x, y, z) = \arctan(\frac{y}{x})$.
Calculate ∇f.

Read the first paragraph of "conservative vector fields" on page 915.
Is $\mathbf{F}(x, y, z) = \frac{(-y,x,0)}{x^2+y^2}$ conservative?

2 Solution

$\nabla f = \frac{(-y,x,0)}{x^2+y^2}$.
The domain of $\mathbf{F}$ is

$$\{(x, y, z) \in \mathbb{R}^3 : \ x^2 + y^2 \neq 0\}.$$

The domain of f is smaller:

$$\{(x, y, z) \in \mathbb{R}^3 : \ x \neq 0\}.$$

Thus, f does NOT define a potential for $\mathbf{F}$.

We'll be able to show $\mathbf{F}$ is not conservative in a week or so. The reason is that the integral

$$\int_{x^2+y^2=1, \ z=0} \mathbf{F} \cdot d\mathbf{r} = 2\pi$$

is non-zero.