Math 31B
Integration and Infinite Series

Midterm 2 Practice

Instructions: You will have 50 minutes to complete the exam.
There will be four questions, worth a total of 48 points.
There are more questions here so that you can practice.
This test is closed book and closed notes. No calculator is allowed.
For full credit show all of your work legibly. Please write your solutions in the space below the questions; INDICATE if you go over the page and/or use scrap paper.
Do not forget to write your name, discussion and UID in the space below.

Name: ____________________
Student ID number: ____________________
Discussion: ____________________

Problem 1.

(a) Let $f(x) = \sqrt{1-x^2} - \cos(\arcsin x)$ and calculate $f'(x)$.

(b) Can you see any significance to your answer in a)?

(c) Calculate the indefinite integral

$$\int \frac{1}{36+25x^2}\, dx.$$

Problem 2.

For (a)-(c), give the value or say, "undefined."

(a) $\tan(\arctan(2)) =$

(b) $\sin(\arcsin(2)) =$

(c) $\arctan(\tan(\frac{7\pi}{3})) =$

(d) Suppose $a \neq 0$.

Calculate the following indefinite integral as I did in class (using a u-substitution and the knowledge of fundamental integrals which relate to inverse trigonometric functions).

$$\int \frac{1}{a^2+x^2}\, dx$$

Problem 3.

(a) Suppose $a > 0$.

Calculate the following definite integral using a u-substitution and the knowledge of fundamental integrals which relate to inverse trigonometric functions.

$$\int_0^{\frac{a}{2}} \frac{1}{\sqrt{a^2-x^2}}\, dx$$

(b) Give a formula for $\sin(\arctan x)$ which does not involve trignometric functions.

(c) Calculate $\lim_{x\to-\infty} \sin(\arctan x)$.

Problem 4.

Calculate

$$\int \frac{2x^2 - 4x + 1}{(x-1)^2(x-2)} dx.$$

Problem 5.

Calculate the following indefinite integral.

$$\int \frac{3x^3 + 6x^2 + 7}{x^2(x^2+7)}\, dx$$

Problem 6.

Calculate the following indefinite integral.

$$\int \frac{6x^3 + 3x^2 + 9x - 8}{(x^2-1)(x^2+4)}\, dx$$

[The numbers have been chosen so that they work out well; they are all whole numbers.

In the method of partial fractions, I found looking at the x^3-coefficient useful.

Problem 7.

Calculate the following antiderivative.

$$\int \frac{4x(x^2 - 2x + 4)}{x^4 - 16}\, dx$$

Help: one of the coefficients in your partial fraction decomposition is 0.

Problem 8.

(a) Let $f(x) = 4 + \frac{3}{1}x + \frac{9}{2}x^2 + \frac{5}{6}x^3 + \frac{11}{24}x^4 + \frac{7}{120}x^5$.
What are the Taylor polynomials $T_3(x)$ and $T_7(x)$ for $f(x)$ centered at 0?

(b) Let $T_n(x) = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \frac{1}{4}(x-1)^4 + \ldots + \frac{(-1)^{n-1}}{n}(x-1)^n$ be the n-th Taylor polynomial for $\ln x$ centered at 1.
Find an n such that
$$\left|\ln\left(\frac{1}{2}\right) - T_n\left(\frac{1}{2}\right)\right| < \frac{1}{10^{10}}.$$

Problem 9.

(a) Let $f(x) = x^3 - 3x^2 + 3x - 1$.
What are the Taylor polynomials $T_2(x)$ and $T_4(x)$ for $f(x)$ centered at 0?

(b) Let $f(x) = x^3 - 3x^2 + 3x - 1$.
What are the Taylor polynomials $T_2(x)$ and $T_4(x)$ for $f(x)$ centered at 1?

(c) Let $T_n(x)$ be the n-th Taylor polynomial for
$$f(x) = 2\cos x + (x-3)^{2017}$$
centered at 0.
Find an n such that $|f(1) - T_n(1)| \leq 1$.

Problem 10.

(a) Assuming that $\lim_{n\to\infty}(1+\frac{1}{n})^n = e$, prove the convergence or divergence of the following series using the ratio test.

$$\sum_{n=1}^{\infty} \frac{n!}{n^n}$$

(b) Prove the convergence or divergence of the following series.

$$\sum_{n=1}^{\infty} (-1)^n \frac{\ln(2^{n^2})}{(2n-1)(2n+1)}$$

Problem 11.

In each case, say whether the series converges absolutely, conditionally, or not at all.

(a) $\sum_{n=1}^{\infty} \frac{(-1)^n n}{\sqrt{n^2+1}}$.

(b) $\sum_{n=2}^{\infty} \frac{(-1)^n}{(\ln n)^2 \sqrt{n}}$.

(c) $\sum_{n=1}^{\infty} \frac{\cos n}{2^n}$.

Problem 12.

For each part of this question:

- *either* give an example to show that the given scenario can arise;
- *or* give a reason why the statement is always false.

(a) The alternating series test applies to say that $\sum_{n=1}^{\infty} a_n$ converges,
and the ratio test is inconclusive.

(b) $\sum_{n=1}^{\infty} a_n$ diverges, and every partial sum s_N is a number between -2 and 2.

(c) $\sum_{n=1}^{\infty} a_n$ is conditionally convergent,
and the ratio test applies and is conclusive.

(d) $\sum_{n=2}^{\infty} a_n$ diverges, each $b_n \neq 0$, $\sum_{n=2}^{\infty} b_n$ converges, and $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$.
(I have not told you that $a_n, b_n > 0$.)