Math 31A
Differential and Integral Calculus

Midterm 2 Practice

Instructions: You have 50 minutes to complete this exam. There are four questions, worth a total of ?? points. This test is closed book and closed notes. No calculator is allowed.

For full credit show all of your work legibly. Please write your solutions in the space below the questions; INDICATE if you go over the page and/or use scrap paper.

Do not forget to write your name, discussion and UID in the space below.

Name: _____
Student ID number: _____
Discussion: _____

Question	Points	Score
1	0	
2	0	
3	0	
4	0	
Total:	0	

Problem 1.

A police car traveling west towards UCLA at a constant speed of 160km/h is in pursuit of a truck traveling north from UCLA at a constant speed of 140km/h.

At time $t = 0$, the police car is 20km east and the truck is 30km north of UCLA.

Calculate the rate at which the distance between the vehicles is changing:

(a) at time $t = 0$;

(b) 5 minutes later.

Problem 2.

A cone has volume $V = \frac{\pi}{3} r^2 h$ and surface area $S = \pi r \sqrt{r^2 + h^2}$. Find the dimensions of the cone with surface area 1 and maximal volume.

Problem 3.

Suppose $f(x) = (x^3 - 3x)^{\frac{1}{3}}$. It is then the case that

$$f'(x) = (x^2 - 1)(x^3 - 3x)^{-\frac{2}{3}} \quad \text{and} \quad f''(x) = -2(x^2 + 1)(x^3 - 3x)^{-\frac{5}{3}}.$$

(a) What are the critical points of $f(x)$?

(b) On what intervals is $f(x)$ increasing/decreasing?

(c) What are the local maxima and minima of $f(x)$?

(d) Describe the concavity of $f(x)$ on the relevant intervals?

(e) Does $f(x)$ have inflection points?

(f) What are $\lim_{x\to\infty} \frac{f(x)}{x}$ and $\lim_{x\to-\infty} \frac{f(x)}{x}$?

(g) Sketch $y = f(x)$.

Problem 4.

Let $f(x) = (\sin x)^2 + \cos x$.

x	$\cos x$	$\sin x$
0	1	0
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	0	1
π	-1	0

(a) Find the critical points of $f(x)$ on $-\frac{\pi}{6} < x < \frac{7\pi}{6}$.

(b) Classify the critical points found in a),
i.e. say whether each is local maximum, local minimum or neither.
You do not need to give the y-values; the x-values will do.
Note: the second derivative test does not fail, so you could try and use it.

(c) Say where $f(x)$ is increasing and decreasing on $0 \leq x \leq \pi$.
No justification is necessary. You'll get full credit for the correct answer.

(d) Find the x-values where $f(x)$ attains its maximum and minimum on $0 \leq x \leq \frac{11\pi}{12}$.
It is not necessary to give the y-values; the x-values will do.
Hint: it might be helpful to calculate $f(\frac{\pi}{2})$ and make use of part c).

(e) Justify the existence of an inflection point between 0 and $\frac{\pi}{2}$.
Warning: don't try to find the x-value or y-value.
If you think you have found it/them then you are either a genius or incorrect.