Math 31A
Differential and Integral Calculus

Final

Instructions: You have 3 hours to complete this exam. There are eight questions, worth a total of ??? points. This test is closed book and closed notes. No calculator is allowed.

For full credit show all of your work legibly. Please write your solutions in the space below the questions; INDICATE if you go over the page and/or use scrap paper.

Do not forget to write your name, discussion and UID in the space below.

Name: _____
Student ID number: _____
Discussion: _____

Question	Points	Score
1	0	
2	0	
3	0	
4	0	
5	0	
6	0	
7	0	
8	0	
Total:	0	

Problem 1.

Calculate each of the limits below.

(a) $\lim_{x\to 4} \frac{x-4}{\sqrt{x}-\sqrt{8-x}}$

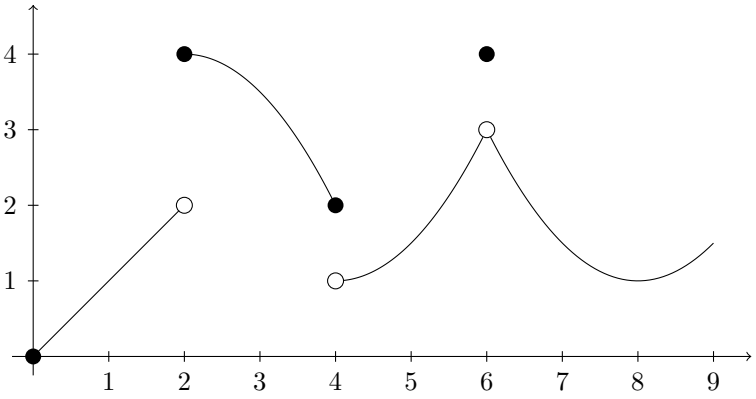
(b) $\lim_{x\to \frac{\pi}{2}} (1-\sin x)\cos(\tan x)$

(c) $\lim_{x\to 3} \frac{x^2-2}{x^2+x-2}$

(d) $\lim_{h\to 0} \frac{\sin(2h)\sin(3h)}{h^2}$

Problem 2.

Let $f(x)$ be the function shown below.



Answer the following questions. (DNE is a possible answer.)

(a) $f'(1) =$

(b) $\lim_{x\to 6} f(x) =$

(c) Is $f(x)$ continuous at $x = 6$?

(d) Is $f(x)$ left continuous, right continuous or neither at $x = 2$?

(e) $\lim_{x\to 2} f(x)f(x+2)$

Problem 3.

(a) Find the tangent line to the graph of

$$x^2 + \sin y = xy^2 + 1$$

at $(1, 0)$.

(b) With the same equation as in part a),
are there any points where the tangent line is horizontal?

Problem 4.

Example 3 of http://tutorial.math.lamar.edu/Classes/CalcI/RelatedRates.aspx

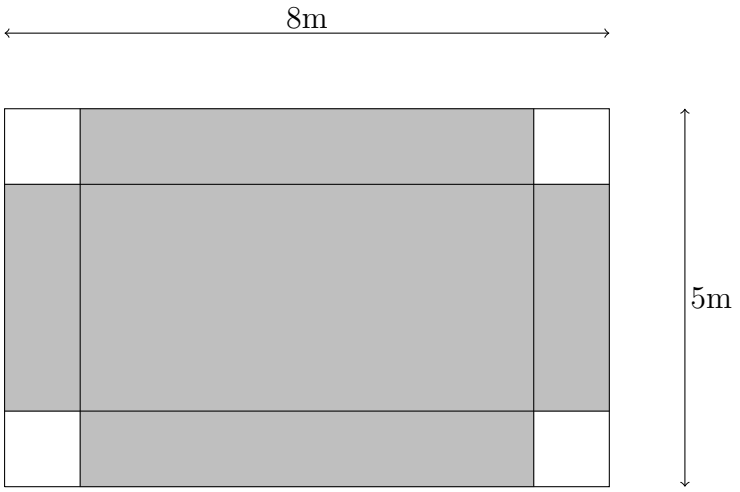
Problem 5.

(a) Let $f(x) = x^3 - 12x$.
Find the global maximum and minimum value of $f(x)$ on the interval $-3 \leq x \leq 5$.

(b) Let $f(x) = \frac{x^4}{4} + 2x^3 - \frac{135}{2}x^2$.
I calculated and factored the first derivative: $f'(x) = (x + 15)x(x - 9)$.
Calculate and classify the critical points of $f(x)$.
Find the inflection points of $f(x)$.
It may help your arithmetic to note that $3 \cdot 3 \cdot 3 \cdot 5 = 135$.

(c) Practice questions from the book with asymptotes too!

Problem 6.



You have an 5 inch by 8 inch piece of cardboard.

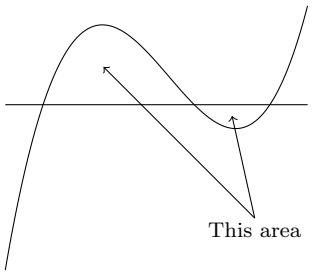
You make a box by cutting out squares from each corner and folding.

How do you build the box in such a way as to maximize volume?

Help! If you have a quadratic that you are struggling to factor, it is always useful to try some values like -2, -1, 0, 1, 2. If plugging in $x = c$ gives zero $(x - c)$ will be a factor.

Problem 7.

Find the area enclosed between the curve $y = 12(x + 2)x(x - 1)$ and the x-axis.



Problem 8.

(a) Calculate the indefinite integral $\int \sec x \tan x(\sec x - 1)dx$.

(b) Calculate the definite integral

$$\int_0^{\frac{\pi}{4}} \sec^4 x \; dx$$

using the substitution $u = \tan x$ and the trigonometric identity $\tan^2 x + 1 = \sec^2 x$.