Math 215, Winter 2014
Midterm 1, January 30

Name: SID:

Instructor: Section:

Instructions
• The total time allowed is 60 minutes.
• The total score is 50 points.
• Use the reverse side of each page if you need extra space.
• Show all your work. A correct answer without intermediate steps will receive no credit.
• Calculators, phones and cheat sheets are not allowed.

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1. Solve the following differential equations for $y(t)$:
   a. $\frac{dy}{dt} = 6t^2 - \frac{y}{t}$, with $y(1) = 1$.

   Solution:

   b. $\frac{dy}{dt} = (y - y^2)te^{t^2}$, with $y(0) = 2$.

   Solution:
2. Match the direction fields with the differential equations by circling A, B, C, or D in each case. Some of these differential equations do not match any of the direction fields - for those cases circle “none”.

(1) \( y' = y^2(2 - y) \)  A   B   C   D   none
(2) \( y' = y(2 - y) \)  A   B   C   D   none
(3) \( y' = y(2 - y)^2 \)  A   B   C   D   none
(4) \( y' = y - 2x \)  A   B   C   D   none
(5) \( y' = y - \sin x \)  A   B   C   D   none
(6) \( y' = y + 2x \)  A   B   C   D   none
3. Consider the following system of differential equations with initial conditions.

\[ \mathbf{x}'(t) = \begin{pmatrix} 4 & 2 \\ 2 & 1 \end{pmatrix} \mathbf{x}(t) \quad \mathbf{x}(0) = \begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} \]

a. Solve the system for \( x_1(0) = 1, x_2(0) = 0. \)

Solution:
3 b. Find all possible initial conditions for the system so that $|\mathbf{x}(t)|$ does not go to infinity as $t \to \infty$. 
4. For this question, assume that turkeys follow Newton’s law of cooling/heating. I start my oven heating at 12pm. The temperature of the oven increases linearly with time until it reaches 150°C at 12.30pm. The oven is initially at room temperature, 20°C.

a. Write an equation for the temperature of the oven as a function of $t$, the time in hours since 12pm.

b. A turkey is placed in the oven at 12pm. The relaxation time of the turkey is 1 hour. (Hint: this means that the constant in Newton’s law is $\pm 1$/hour depending on your sign convention). The temperature of the turkey at 12pm is 4°C. Find the temperature of the turkey at 12.30pm (leave powers of $e$ as part of your answer).