Don’t worry - just do your best :-) 

1. The marks you receive during the term will likely be scaled based on the class results on the final exam. Even if it seems difficult, that doesn’t mean the term marks will be low!

2. For questions 2 – 5, generous marks may be given for honest attempts, even if you get stuck - provided you show your work.

Instructions and policies

1. Please print your name, student number, and provide your signature right away, before you attempt any of the problems. Check that this booklet contains eleven (11) pages.

2. All your work must be shown for full credit on questions 2 – 5. No work is required in question 1, and a correct answer will receive 3 points; partial credit of 1 point may be given in question 1 if the answer you provide is wrong.

3. Unless a question specifies otherwise, you do not need to simplify your answers.

4. No calculators, electronic devices, or formula sheets are allowed during the test.

5. Ten (10) minutes before the end of the test period you will be given a verbal notice. After that time, you must remain seated until all test papers have been collected.

6. Exposing your test paper to others or looking at the paper of another student, even if unintentional, will be deemed as cheating and subject to disciplinary action.

7. When the test period is over, you will be instructed to put away writing implements. Put away all pens and pencils at this point. Continuing to write past this instruction will be considered cheating. Please remain seated and pass your test paper down the row to the nearest indicated aisle. You may leave your seat only after all the tests have been collected.

8. A certain portion of the tests will be randomly selected and photocopied. This is a warning in case an individual modifies their answers and asks for a re-grading, which would be cheating.
1 Short Answer (15 marks: 3 marks per part)

   a Find the length of the curve $r = \sin(\theta)$ from $\theta = 0$ to $\theta = \pi/3$.

   b Write down the Trapezoid rule approximation $T_4$ for evaluating $\int_{-1}^{1} \frac{dx}{2-x^4}$.

   c Find the first three nonzero terms in the Maclaurin series for $\arcsin(2x)$ in powers of $x$. 
d  Evaluate \( \int_{-1}^{1} x^3 \sqrt{\frac{1+4x^2}{1+x^4}} \, dx \).

e  A tank in the form of a rectangular prism of base area 4 ft\(^2\) and height 10 ft is completely filled with water. Approximating the weight of water to be 60 lbs ft\(^{-3}\), find the work done in pumping all the water out of the top of the tank. Simplify your answer as much as possible.
2 Long Answer (10 marks)

Find the volume of the described solid \( S \): the base of \( S \) is an elliptical region with boundary curve \( 9x^2 + 4y^2 = 36 \). Cross-sections perpendicular to the \( x \)-axis are isosceles right triangles with hypotenuse in the base.
3 Long Answer (10 marks)

Find

a (5 marks) \( \int x^2 \tan^{-1} x \, dx \)

b (5 marks) \( \int_{0}^{\pi/4} \frac{\cos \theta \, d\theta}{(1 - \sin \theta)^{2/3}} \)
4 Long Answer (10 marks)

In this problem you may (or may not) find the following facts useful:

\[ \lim_{x \to \infty} \frac{\ln x}{x^a} = 0 \text{ for } a > 0 \]
\[ \lim_{x \to 0^+} x^a \ln x = 0 \text{ for } a > 0 \]
\[ \int \frac{dx}{\sqrt{x^2 + 1}} = \ln |\sqrt{x^2 + 1} + x| + C \]

a (2 marks) Show that \( \int f(x) \ln x \, dx = F(x) \ln x - \int \frac{F(x)}{x} \, dx \) where \( F'(x) = f(x) \).

b (4 marks) Show that \( \int_1^\infty \frac{\ln x}{x^p} \, dx \) converges for \( p > 1 \) and diverges for \( p \leq 1 \).
c (2 marks) Does $\int_0^1 \frac{\ln x}{\sqrt{x}} \, dx$ converge?

d (2 marks) For which values of $s$ does $\int_0^\infty \frac{\ln x}{\sqrt{x}^s+1} \, dx$ converge?
5 Conceptual Question (5 marks)

Consider a smooth, differentiable cartesian curve $y = f(x)$ for $x \in [a, b]$.

a (3 marks) By breaking $[a, b]$ into $n$ subintervals of equal width, come up with a Riemann sum approximation to the arc length by approximating $f$ as a straight line over each subinterval.

b (2 marks) Derive the arc length formula from your Riemann sum.
Figure 1: Perhaps the greatest XKCD comic ever - hope you enjoy!

Bonus a (1 mark)

A great philosophical question: why should you get a bonus mark?

Bonus b (1.5 marks)

Evaluate \( \int_{0}^{\pi/2} \frac{d\theta}{1+(\tan \theta)^{\sqrt{2}}} \). Hint: try a substitution with \( \psi = \frac{\pi}{2} - \theta \).
Mark breakdown:

<table>
<thead>
<tr>
<th>Question</th>
<th>Marks out of</th>
</tr>
</thead>
<tbody>
<tr>
<td>Question 1</td>
<td>out of 15</td>
</tr>
<tr>
<td>Question 2</td>
<td>out of 10</td>
</tr>
<tr>
<td>Question 3</td>
<td>out of 10</td>
</tr>
<tr>
<td>Question 4</td>
<td>out of 10</td>
</tr>
<tr>
<td>Question 5</td>
<td>out of 5</td>
</tr>
<tr>
<td>Bonus</td>
<td>out of 2.5</td>
</tr>
<tr>
<td>TOTAL</td>
<td>out of 50</td>
</tr>
</tbody>
</table>

11