(1) Show that $A(1)$ is an 8-dimensional algebra over $F_2$ generated by $1$, $Sq^1$, $Sq^2$, $Sq^3$, $Sq^2Sq^1$, $Sq^2Sq^2$, $Sq^3Sq^1$, and $Sq^3Sq^3$.

(2) Let $HZ$ be the $A(1)$-module $A(1) \otimes_{A(0)} F_2$, where $A(0)$ is the subalgebra generated by $Sq^1$. Compute $Ext_{A(1)}^{s,t}(HZ, F_2)$ for all $s$ and $t$. Do this two ways: first by actually computing a minimal resolution and second by using a change-of-rings.

(3) Show that if $P_*$ is a minimal resolution of $M$, then $Ext_{A(1)}^{s,t}(M, F_2)$ is just $Hom_{A(1)}(P_s, F_2)$.

(4) Let $C(\eta)$ be the $A(1)$-module generated by a class $a$ subject to the relations $Sq^1(a) = 0 = Sq^2(a)$. Show that as an $F_2$-vector space, $C(\eta)$ is two dimensional, and compute $Ext_{A(1)}(C(\eta), F_2)$.

(5) We saw that the cohomology of the integral Eilenberg-MacLane spaces was characterized by the additional relation: $Sq^1(\iota_n) = 0$. This infinitely deloops to show us that $H^*(HZ) = A \otimes_{A(0)} F_2$. Use this and the Adams spectral sequence to show that the class corresponding to $Sq^1$ in $Ext^{1,1}$ detects multiplication by 2.

(6) Building on the previous problem, run the Adams spectral sequence for $HZ/2^k$ as $k \geq 1$. You should see that the $E_2$ terms are all isomorphic for $k > 1$, and the only difference is in the differentials. You may use the fact that a short exact sequence of modules induces a long exact sequence in $Ext$. 