PROBLEM SET THE LAST

The first few problems are from the book. I'll list them via number in the book plus the actual statement of the problem.

These are from Chapter 9.

4. Prove the parallelogram law.
5. Prove Appollonius' identity:

$$||w-u||^2 + ||w-v||^2 = \frac{1}{2}||u-v||^2 + 2\left\|w - \frac{1}{2}(u+v)\right\|^2.$$

7. Prove that two vectors u and v in an real inner product space V are orthogonal if and only if

$$||u+v||^2 = ||u||^2 + ||v||^2.$$

8. Show that an isometry is injective.
15. Let V be a finite dimensional inner product space. Prove that for any subset X of V, $X^{\perp\perp} = \langle X \rangle$.
16. Let P_3 be the inner product space of all polynomials of degree at most 3, under the inner product

$$\langle p(x), q(x) \rangle = \int_{-\infty}^{\infty} p(x)q(x)e^{-x^2}\,dx.$$

Apply Gram-Schmidt to the basis $\{1, x, x^2, x^3\}$.

From Chapter 11

2a. Use the triangle inequality to prove

$$|d(x,y) - d(a,b)| \leq d(x,a) + d(y,b).$$

2b. Prove that

$$|d(x,z) - d(y,z)| \leq d(x,y).$$

27. Suppose that (x_n) is a Cauchy sequence in a metric space M and that some subsequence (x_{n_k}) of (x_n) converges to x. Prove that (x_n) converges to x.
36. Show that the metric spaces $C[a,b]$ and $C[c,d]$, under the sup metric, are isometric (hint: rescale).

Now a fun one.

Use the Fourrier series for x^3 and Parseval's equality to show

$$\sum_{n\geq 1} \frac{1}{n^6} = \frac{\pi^6}{945}.$$